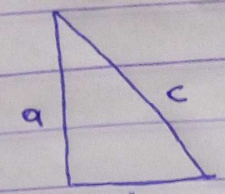
Onyeuku Ugochukwu Promise
17/ENG04/061
Elect/Elect

Limits

The hypotenuse of a right angle triangle is denoted by 'c' and the other two sides are denoted as 'a and b' is ±1.5% find the maximum possible error in calculating
- The length of the hypotenuse c

solu



$$\text{Area} = \frac{1}{2} \cdot b \cdot h = \frac{1}{2} \cdot b \cdot a = \frac{b \cdot a}{2}$$

$$\frac{\delta A}{\delta a} = \frac{b}{2}, \quad \frac{\delta A}{\delta b} = \frac{a}{2}$$

$$\therefore \delta A = \frac{\delta A}{\delta a} \cdot \delta a + \frac{\delta A}{\delta b} \cdot \delta b$$

$$\delta A = \frac{b}{2} \cdot \frac{1.5}{100} a + \frac{a}{2} \cdot \frac{1.5}{100} \cdot b$$

$$= \frac{b}{2} \times \frac{\pm 1.5a}{100} + \frac{a}{2} \times \frac{\pm 1.5b}{100}$$

$\pm 1.5\% = (3/2 \div 100)\, b$

$= 3/200$

$$\frac{b}{2}\left(\pm \frac{3b}{200}\right) + \frac{a}{2}\left(\pm \frac{3a}{200}\right)$$

$$= \pm \frac{a \cdot b}{2}\left[\frac{3}{200} + \frac{3}{200}\right]$$

$$= \pm \frac{a \cdot b}{2}\left(\frac{3}{200} + \frac{3}{200}\right)$$

$$= \pm A\left(\frac{3}{100}\right)$$

resurfinge

$$= \frac{\pm 1.5ab}{200} + \frac{(\pm 1.5ab)}{100}$$

$$= ab\left(\frac{3ab}{200}\right)$$

$$\therefore \Delta A = \pm 3\% \text{ of } A \quad (\delta A = \pm 3\%)$$

$c = \sqrt{a^2+b^2} = (a^2+b^2)^{1/2}$

$\therefore \frac{dc}{da} = \frac{1}{2}(a^2+b^2)^{-1/2} \cdot 2a \left(\frac{dc}{du} \times \frac{du}{da}\right)$

$u = (a^2+b^2)$

$\frac{du}{da} = 2a$

$\frac{dc}{du} = \frac{u^{-1/2}}{2}$

$\therefore \frac{dc}{da} = \frac{1}{2}(a^2+b^2)^{-1/2} \cdot 2a$

$\therefore d\,dc/db = 1/2(a^2+b^2)^{-1/2} \cdot 2b = b/\sqrt{a^2+b^2}$

$= \frac{(a^2+b^2)^{-1/2}}{2} \cdot 2a$

$da = \frac{\pm 3a}{200}$, $db = \frac{\pm 3b}{200}$

$dc = \frac{dc}{da} \cdot \delta a + \frac{\delta c}{\delta b} \cdot \delta b$

$= \left(\frac{a}{\sqrt{a^2+b^2}} \cdot \frac{\pm 3a}{200}\right) + \left(\frac{b}{\sqrt{a^2+b^2}} \cdot \frac{\pm 3b}{200}\right)$

$= \frac{3}{200}\left(\frac{a^2+b^2}{\sqrt{a^2+b^2}}\right)$

$= \frac{3}{200}\left(\frac{c^2}{c}\right)$

$= \frac{3}{200}(c)$

$dc = 1.5\%$ of c