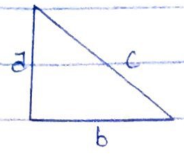
1/10/18 FIDE-AKWUOBI ANTHONY CHIZALU 17/ENG06/037 ENG 281 MECHANICAL ENGINEERING

The hypotenuse of a right angled triangle is denoted as c, and the other two sides as a and b. If the possible error of measuring each of a and b is ±1.5%, find the maximum possible error in calculating: (a) the area of the triangle (b) the length of the hypotenuse.

Solution



(a) Recall area of a right angled triangle $A = \frac{1}{2}ab$

$$\therefore \delta A = \frac{\partial A}{\partial a}\cdot \delta a + \frac{\partial A}{\partial b}\cdot \delta b$$

$$\frac{\partial A}{\partial a} = \frac{1}{2}b, \quad \frac{\partial A}{\partial b} = \frac{1}{2}a \quad \therefore \delta A = \frac{b}{2}\cdot \delta a + \frac{a}{2}\cdot \delta b$$

Error $= \pm 1.5\%$

$$\therefore \delta a = \frac{\pm 1.5}{100}a, \quad \delta b = \frac{\pm 1.5}{100}b$$

$$\therefore \delta A = \frac{b}{2}\cdot \frac{\pm 1.5a}{100} + \frac{a}{2}\cdot \frac{\pm 1.5b}{100}$$

$$\delta A = \frac{ab}{2}\left(\frac{1.5}{100} + \frac{1.5}{100}\right)$$

$$\delta A = \frac{ab}{2}\left(\frac{1.5+1.5}{100}\right) = \left(\frac{3}{100}\right)$$

$$\therefore \delta A = \frac{ab}{2}\left(\frac{\pm 3}{100}\right)$$

Recall; $A = \frac{ab}{2}$ $\therefore$ Replacing it $\therefore \delta A = A\left(\frac{\pm 3}{100}\right)$

$\therefore$ Maximum possible error in calculating A is $\pm 3\%$

(b) Recall hypotenuse $c^2 = a^2 + b^2$

$$\therefore c = \sqrt{a^2 + b^2} \quad \therefore c = (a^2 + b^2)^{\frac{1}{2}}$$

$$\delta c = \frac{\partial c}{\partial a}\cdot \delta a + \frac{\partial c}{\partial b}\cdot \delta b$$

$\therefore$ For $\frac{\partial c}{\partial a}$: $c = (a^2 + b^2)^{\frac{1}{2}}$ $\therefore$ Let $a^2 + b^2 = u$ $\therefore c = u^{\frac{1}{2}}$

$$\therefore \frac{\partial u}{\partial a} = 2a, \quad \frac{\partial c}{\partial u} = \frac{1}{2}u^{-\frac{1}{2}}$$

$\therefore \frac{\partial C}{\partial a} = \frac{\partial u}{\partial a} \times \frac{\partial C}{\partial u} \Rightarrow \cancel{2}a \cdot \frac{1}{\cancel{2}} u^{-\frac{1}{2}}$

$$\Rightarrow \frac{a}{\sqrt{u}}$$

Recall $u = a^2 + b^2$

$$\therefore \frac{\partial C}{\partial a} = \frac{a}{\sqrt{a^2 + b^2}}$$

For $\frac{\partial C}{\partial b} \Rightarrow$ Let $u = a^2 + b^2 \quad \therefore C = u^{\frac{1}{2}}$

$$\therefore \frac{\partial C}{\partial u} = \frac{1}{2} u^{-\frac{1}{2}} \quad \therefore \frac{\partial u}{\partial b} = 2b$$

$$\frac{\partial C}{\partial b} = \frac{\partial C}{\partial u} \times \frac{\partial u}{\partial b} \quad \therefore \frac{\partial C}{\partial b} = \cancel{2}b \cdot \frac{1}{\cancel{2}} u^{-\frac{1}{2}} = \frac{b}{\sqrt{u}}$$

$\therefore$ Recall $u = a^2 + b^2 \quad \therefore \frac{\partial C}{\partial b} = \frac{b}{\sqrt{a^2 + b^2}}$

$$\therefore \delta C = \frac{a}{\sqrt{a^2 + b^2}} \cdot \delta a + \frac{b}{\sqrt{a^2 + b^2}} \cdot \delta b \quad \therefore \delta a = \frac{\pm 1.5 a}{100}, \ \delta b = \frac{\pm 1.5 b}{100}$$

$$\therefore \delta C = \frac{a}{\sqrt{a^2 + b^2}} \left(\frac{1.5a}{100}\right) + \frac{b}{\sqrt{a^2 + b^2}} \left(\frac{1.5b}{100}\right)$$

$$\therefore \delta C = \frac{a^2}{\sqrt{a^2 + b^2}} \left(\frac{\pm 3}{200}\right) + \frac{b^2}{\sqrt{a^2 + b^2}} \left(\frac{\pm 3}{200}\right)$$

$$\Rightarrow \frac{\pm 3}{200} \left[\frac{a^2}{\sqrt{a^2 + b^2}} + \frac{b^2}{\sqrt{a^2 + b^2}}\right]$$

$$\frac{\pm 3}{200} \left[\frac{a^2 + b^2}{\sqrt{a^2 + b^2}}\right]$$

$$\frac{\pm 3}{200} \left[\frac{a^2 + b^2}{(a^2 + b^2)^{\frac{1}{2}}}\right]$$

Through indices $\therefore \frac{\pm 3}{200} \left[(a^2 + b^2)^{1 - \frac{1}{2}}\right]$

$$\Rightarrow \frac{\pm 3}{200} (a^2 + b^2)^{\frac{1}{2}}$$

Recall $C = (a^2 + b^2)^{\frac{1}{2}}$ or $\sqrt{a^2 + b^2}$

$$\Rightarrow \frac{\pm 3}{200} \text{ or } 1.5\% \text{ of } C$$

$\therefore$ <u>Maximum percentage error $= \pm 1.5\%$ of C</u>