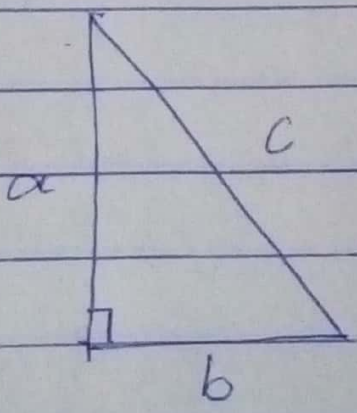
ESSANG ROSEMARY
17/ENG04/023 ELECTRICAL/ELECTRONICS ENGINEERING
~~ELECTRONICS~~

ENG 281

The hypotenuse of a right-angled triangle is denoted as the other 2 sides are denoted as a and b. If the possible % measuring each of a and b is ± 1.5%, find the maximum error in calculating:

i) the area of the triangle
ii) the length of the hypotenuse

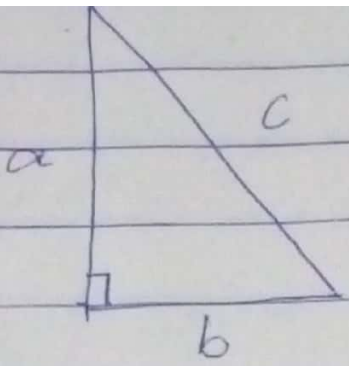
Solution



i) $A = \frac{ba}{2}$ $A = f(b, a)$

$$\delta_A = \frac{\partial A}{\partial b}\delta b + \frac{\partial A}{\partial a}\delta a$$

$$\frac{\partial A}{\partial b} = \frac{a}{2}, \quad \frac{\partial A}{\partial a} = \frac{b}{2}$$



i) $A = \frac{ba}{2}$ $\quad A = f(b, a)$

$$\delta_A = \frac{\partial A}{\partial b}\delta b + \frac{\partial A}{\partial a}\delta a$$

$$\frac{\partial A}{\partial b} = \frac{a}{2}, \quad \frac{\partial A}{\partial a} = \frac{b}{2}$$

$$\delta b = \pm 1.5\, b, \quad \delta a = \pm 1.5\, a$$

$$\delta_A = \frac{a}{2}(\pm 1.5\% b) + \frac{b}{2}(\pm 1.5\% a)$$

$$= \frac{ba}{2}(\pm 1.5\%) + \frac{ba}{2}(\pm 1.5\%)$$

$$= \frac{ba}{2}[\pm 1.5\% + \pm 1.5\%]$$

$$\delta_A = \frac{ba}{2}[\pm 3.0\%] = \pm 3\% A$$

(ii) $C^2 = a^2 + b^2$

Square root both sides

$\sqrt{C^2} = \sqrt{a^2 + b^2}$

$C = f(a, b)$

$$\delta C = \frac{\partial C}{\partial a} \delta a + \frac{\partial C}{\partial b} \delta b$$

Let $u = a^2 + b^2$

then $C = \sqrt{u}$

$$\frac{\partial C}{\partial a} = \frac{\partial C}{\partial u} \times \frac{\partial u}{\partial a}$$

$$\frac{\partial C}{\partial u} = \frac{1}{2} u^{-1/2} \qquad \frac{\partial u}{\partial a} = 2a$$

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$$= \frac{2a}{2} = a$$

Let $u = a^2 + b^2$

Then $c = \sqrt{u}$

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$$\frac{\partial c}{\partial a} = \frac{1}{2} u^{-1/2} \times 2a$$

$$= \frac{2a}{2\sqrt{u}} = \frac{2a}{2\sqrt{a^2+b^2}}$$

$$= \frac{a}{\sqrt{a^2+b^2}}$$

$$\frac{\partial c}{\partial b} = \frac{\partial c}{\partial u} \times \frac{\partial u}{\partial b}$$

$$= \frac{1}{2} u^{-1/2} \times 2b$$

$$= \frac{2b}{2\sqrt{u}} = \frac{2b}{2\sqrt{a^2+b^2}}$$

$$= \frac{b}{\sqrt{a^2+b^2}}$$

$$\delta c = \frac{a}{\sqrt{a^2+b^2}} \times (\pm 1.5\% a) + \frac{b}{\sqrt{a^2+b^2}} \times (\pm 1.5\% b)$$

$$= \pm 1.5\% \left[\frac{a^2}{\sqrt{a^2+b^2}} + \frac{b^2}{\sqrt{a^2+b^2}} \right]$$

$$= \pm 1.5\% \left[\frac{a^2+b^2}{\sqrt{a^2+b^2}} \right]$$

but $c = \sqrt{a^2+b^2}$

and $c^2 = a^2+b^2$

$$= \pm 1.5\% \left[\frac{c^2 c}{c} \right]$$

$$\delta c = \pm 1.5\% c$$