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ENG 281

The hypotenuse of a right-angled triangle is denoted as c, and the other two sides are denoted as a and b. If the possible error of measuring each of a and b is ±1.5% . find the maximum possible error in calculating:
a) the area of the triangle, and
b) the length of the hypotenuse.

Solution

$$A = \frac{1}{2}ab = \frac{ab}{2}$$

$$\delta A = \frac{\delta A}{\delta a}\delta a + \frac{\delta A}{\delta b}\delta b$$

$$\frac{\delta A}{\delta a} = \frac{b}{2}, \quad \frac{\delta A}{\delta b} = \frac{a}{2}$$

$$\delta A = \frac{b}{2}\left(\frac{1.5}{100}a\right) + \frac{a}{2}\left(\frac{1.5}{100}b\right)$$

$$\delta A = \frac{b(0.015a)}{2} + \frac{a(0.015b)}{2}$$

$$\frac{(ab)(0.015)}{2} + \frac{(ab)(0.015)}{2}$$

$$= \frac{ab}{2}\pm(0.015 + 0.015) = \frac{ab}{2}(\pm 0.03)$$

$$A = \frac{ab}{2} \quad \therefore$$

$$\delta A = \pm A\, 0.03$$

$\delta A = \pm 3$ per cent of A.

b. $h = \sqrt{a^2 + b^2}$

$= (a^2 + b^2)^{\frac{1}{2}}$

$$\delta h = \frac{\delta h}{\delta a} \delta a + \frac{\delta h}{\delta b} \delta b$$

$$\frac{\delta h}{\delta a} = \frac{1}{2}(a^2 + b)^{-\frac{1}{2}}(2a), \quad \frac{\delta h}{\delta b} = \frac{1}{2}(a^2 + b^2)^{-\frac{1}{2}}(2b)$$

$$\frac{\delta h}{\delta a} = \frac{1}{\cancel{2}}(a^2 + b)^{-\frac{1}{2}}(\cancel{2}a) = \frac{a}{\sqrt{(a^2 + b^2)}}$$

$$\frac{\delta h}{\delta a} = \frac{1}{\cancel{2}}(a^2 + b^2)^{-\frac{1}{2}}(\cancel{2}b) = \frac{b}{\sqrt{a^2 + b^2}}$$

$$\delta h = \frac{a}{\sqrt{a^2 + b^2}}\left(\frac{\pm 1.5}{100} a\right) + \frac{b}{\sqrt{a^2 + b^2}}\left(\frac{\pm 1.5}{100} b\right)$$

$$\pm 0.015\left(\frac{a^2}{\sqrt{a^2 + b^2}}\right) + \pm 0.015\left(\frac{b^2}{\sqrt{a^2 + b^2}}\right)$$

$$\pm 0.015\left(\frac{a^2}{\sqrt{a^2 + b^2}} + \frac{b^2}{\sqrt{a^2 + b^2}}\right)$$

$$\pm 0.015\left(\frac{a^2 + b^2}{\sqrt{a^2 + b^2}}\right)$$

where $h = \sqrt{a^2 + b^2}$

$h^2 = a^2 + b^2$

$$\pm 0.015\left(\frac{h^2}{h}\right)$$

$$\delta h = \pm 0.015 h$$

$\delta h = \pm 0.015$ per cent of h,