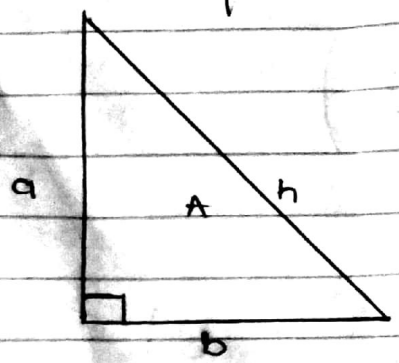
Name: ALEGBELEYE OLUWAFEMI OLADIPUPO
Department: ELECTRICAL/ELECTRONICS
Matric Number: 17/ENG04/011
Signature:

1. The hypotenuse of a right-angled triangle is denoted as c, and the other two sides are denoted as a and b. If the possible error of measuring each a and b is ±1.5% find the maximum possible error in calculating:

a) the area of triangle, and
b) the length of the hypotenuse.




Solution.

$$A = \frac{1}{2}ab = \frac{ab}{2}$$

$$\delta A = \frac{\partial A}{\partial a}\delta a + \frac{\partial A}{\partial b}\delta b$$

$$\frac{\partial A}{\partial a} = \frac{b}{2} \qquad \frac{\partial A}{\partial b} = \frac{a}{2}$$

$$\delta a = \pm\frac{1.5}{100} = \pm\frac{3}{200} \qquad \delta b = \pm\frac{1.5}{100} = \pm\frac{3}{200}$$

$$\delta A = \left(\frac{b}{2}\right)\cdot\left(\pm\frac{3a}{200}\right) + \left(\frac{a}{2}\right)\cdot\left(\pm\frac{3b}{200}\right)$$

$$\delta A = \frac{\pm ab}{2}\left[\frac{3}{200} + \frac{3}{200}\right] = \pm A\left[\frac{3}{100}\right]$$

$$\delta A = \pm A\,\frac{3}{100}$$

$$\therefore \delta A = 3 \text{ per cent of } A$$

$h = \sqrt{a^2+b^2} = (a^2+b^2)^{1/2}$ and $h = f(a,b)$

$$\sigma h = \frac{\partial h}{\partial a}\sigma a + \frac{\partial h}{\partial b}\sigma b$$

$$\frac{\partial h}{\partial a} = a(a^2+b^2)^{-1/2} = \frac{a}{\sqrt{a^2+b^2}}$$

$$\frac{\partial h}{\partial b} = b(a^2+b^2)^{-1/2} = \frac{b}{\sqrt{a^2+b^2}}$$

$$\sigma a = \left(\pm\frac{1.5}{100}\right) \cdot \left(\pm\frac{3}{200}\right)$$

$$\sigma b = \left(\pm\frac{1.5}{100}\right) \cdot \left(\pm\frac{3}{200}\right)$$

$$\sigma h = \frac{a}{\sqrt{a^2+b^2}} \cdot \left(\pm\frac{3a}{200}\right) + \frac{b}{\sqrt{a^2+b^2}} \cdot \left(\pm\frac{3b}{200}\right)$$

$$= \pm\frac{a^2}{\sqrt{a^2+b^2}} \cdot \left(\pm\frac{3}{200}\right) + \frac{b^2}{\sqrt{a^2+b^2}}\left(\pm\frac{3}{200}\right)$$

$$= \pm\frac{3}{200}\left(\frac{a^2+b^2}{\sqrt{a^2+b^2}}\right)$$

$$= \pm\frac{3}{200}\sqrt{a^2+b^2}$$

Recall $\sqrt{a^2+b^2} = \left(\frac{a^2+b^2}{\sqrt{a^2+b^2}}\right)$

$$= \pm\frac{3}{200}(h)$$

$\therefore \sigma h = 0.015$ per cent of $h$