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Dept : Mechanical Engineering

Matric no: 17/ENR06/087


Level : 200L

Assignment

The hypotenuse of a right-angled triangle is denoted as c , and the other two sides are denoted as a and b . If the possible error of measuring each of a and b is $\pm 1.5\%$. Find the maximum possible error in calculating:

→ the area of the triangle and

→ the length of the hypotenuse.

Sol. Recall, $A = \frac{1}{2}ab$ 

$$\rightarrow \Delta A = \frac{\partial A}{\partial a} \cdot \Delta a + \frac{\partial A}{\partial b} \cdot \Delta b$$

$$\frac{\partial A}{\partial a} = \frac{1}{2} \cdot b = \frac{b}{2}, \quad \frac{\partial A}{\partial b} = \frac{1}{2} \cdot a = \frac{a}{2}$$

$$\Delta a = \pm 1.5/100 a, \quad \Delta b = \pm 1.5/100 b$$

$$= \pm \frac{3a}{200}, \quad = \pm \frac{3b}{200}$$

$$\Delta A = \frac{\partial A}{\partial a} \cdot \Delta a + \frac{\partial A}{\partial b} \cdot \Delta b$$

$$= \frac{b}{2} \times \pm \frac{3a}{200} + \frac{a}{2} \times \pm \frac{3b}{200}$$

$$= \frac{ab}{2} \left[\pm \frac{3}{200} + \frac{3}{200} \right] = \frac{ab}{2} = \frac{3}{100} = \frac{ab}{2} \pm \frac{3}{100} = \underline{\underline{3\%}}$$

$$\rightarrow \text{Recall, } h = \sqrt{a^2 + b^2} = (a^2 + b^2)^{1/2}$$

$$\text{Let } u = a^2 + b^2, \quad \frac{\partial h}{\partial u} = \frac{1}{2} u^{-1/2}, \quad \frac{\partial u}{\partial a} = 2a, \quad \frac{\partial u}{\partial b} = 2b$$

$$\frac{\partial h}{\partial a} = \frac{\partial h}{\partial u} \times \frac{\partial u}{\partial a} = \frac{1}{2} u^{-1/2} \times 2a = \frac{a}{\sqrt{a^2 + b^2}}$$

$$\frac{\partial h}{\partial b} = \frac{\partial h}{\partial u} \times \frac{\partial u}{\partial b} = \frac{1}{2} u^{-1/2} \times 2b = \frac{b}{\sqrt{a^2 + b^2}}$$

$$\Delta h = \frac{\partial h}{\partial a} \cdot \Delta a + \frac{\partial h}{\partial b} \cdot \Delta b$$

$$\Delta a = \pm \frac{3a}{200}, \quad \Delta b = \pm \frac{3b}{200}$$

$$\Delta h = \frac{a}{\sqrt{a^2 + b^2}} \times \pm \frac{3a}{200} + \frac{b}{\sqrt{a^2 + b^2}} \times \pm \frac{3b}{200}$$

$$\Delta h = \frac{a^2 + b^2}{\sqrt{a^2 + b^2}} \left[\pm \frac{3}{200} \right] = \frac{3}{200} = \underline{\underline{1.5\%}}$$