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17/ENG02/1039
Computer Engineering.

Question

The hypotenuse of a right-angled triangle is denoted as "C", and the other two sides are denoted as a and b. If the possible error of measuring each side of a and b is $\pm 1.5\%$, find the maximum possible error in calculating.

(a) The area of the triangle
(b) The length of the hypotenuse.

Soln

(a) Let Y be the are of the Δ The area

Area of $\Delta = 1/2\ b \times h$

$\therefore$ Area, $Y = 1/2 \times b \times a = ab/2$

Applying the formula $\delta Y = \frac{\partial Y}{\partial a} \cdot \delta a \times \frac{\partial Y}{\partial b} \cdot \delta b$

$\frac{\partial Y}{\partial a} = \frac{b}{2}$, $\frac{\partial Y}{\partial b} = a/2$

$\delta a = \pm \frac{1.5}{100}$ of $a = \pm \frac{1.5a}{100}$, $\delta b = \pm \frac{1.5}{100}$ of $b = \pm \frac{1.5b}{100}$

$\delta Y = b/2 \left(\frac{\pm 1.5a}{100}\right) + a/2 \left(\frac{\pm 1.5b}{100}\right)$

$\delta Y = \frac{ab}{2}\left(\frac{\pm 1.5}{100}\right) \pm \frac{ab}{2}\left(\frac{\pm 1.5}{100}\right)$

$\delta Y = \pm \frac{ab}{2}\left(\frac{1.5}{100}\right) + \pm \frac{ab}{2}\left(\frac{1.5}{100}\right)$

$\delta Y = \pm \frac{ab}{2}\left(\frac{1.5 + 1.5}{100}\right)$

$\delta Y = \pm \frac{ab}{2}\left(\frac{3}{100}\right)$ Solve $Y = \frac{ab}{2}$, $\delta Y = \pm A \left(\frac{3}{100}\right)$

Therefore the maximum possible error in calculating the area of the triangle $\delta Y = 3$ percent (3%) of the area of the triangle (3% of Y)

(b) The length

From Pythagoras theorem

$$c^2 = a^2 + b^2, \quad c = \sqrt{a^2+b^2} \text{ or } (a^2+b^2)^{1/2}$$

using the formula

$$\delta c = \frac{\partial c}{\partial a}\delta a + \frac{\partial c}{\partial b}\delta b$$

$\frac{\partial c}{\partial a}$ using the formula for function of function

Let $u = a^2+b^2$, $\frac{\partial u}{\partial a} = 2a$, $\frac{\partial c}{\partial a} = \frac{1}{2}(u)^{-1/2} = \frac{1}{2}(a^2+b^2)^{-1/2}$

$$\frac{\partial c}{\partial a} = \frac{1}{2}(a^2+b^2)^{-1/2} \times (2a) = \frac{2a(a^2+b^2)^{-1/2}}{2} = a(a^2+b^2)^{-1/2} \text{ or } \frac{a}{\sqrt{a^2+b^2}}$$

$\frac{\partial c}{\partial b}$ using formula for function of a function of a function

$$\left(\frac{\partial c}{\partial b} = \frac{\partial c}{\partial u} = \frac{\partial u}{\partial b}\right)$$

Let $u = a^2+b^2$, $\frac{\partial u}{\partial b} = 2b$, $\frac{\partial c}{\partial u} = \frac{1}{2}(u)^{-1/2} = \frac{1}{2}(a^2+b)^{-1/2}$

$$\frac{\partial c}{\partial b} = \frac{1}{2}(a^2+b^2)^{-1/2} \times (2b) = \frac{2b(a^2+b^2)^{-1/2}}{2} = b(a^2+b^2)^{-1/2} \text{ or } \frac{b}{\sqrt{a^2+b^2}}$$

From eq(a)

$$\delta a = \frac{\pm 1.5a}{100} \text{ and } \delta b = \pm\frac{1.5b}{100}$$

$$\therefore \delta c = \frac{a}{\sqrt{a^2+b^2}}\left(\frac{\pm 1.5a}{100}\right) + \frac{b}{\sqrt{a^2+b^2}}\left(\pm\frac{1.5b}{100}\right)$$

$$\delta c = \frac{\pm 1.5a^2}{(100)(\sqrt{a^2+b^2})} + \frac{1.5b^2}{100(\sqrt{a^2+b^2})}$$

$$\delta c = \frac{\pm 1.5 (a^2 + b^2)}{(100)(\sqrt{a^2 + b^2})}$$

$$\delta c = \frac{\pm 1.5}{100} (\sqrt{a^2 + b^2})$$

$$\delta c = \frac{\pm 1.5}{100}(c)$$

Therefore the maximum possible error on calculating the length of the hypotenuse, δc = 1.5 percent of the hypotenuse (1.5% of c).