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Civil Engineering

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ENG 281

Question

The hypotenuse of a right-angled triangle is denoted as c and the other two sides are denoted as a and b . If the possible error of measuring each a and b is $\pm 1.5\%$. Find the maximum possible error in calculating:

- Area of the triangle
- The length of the hypotenuse

Answer

a) Let the area of the triangle be A

$$A = \frac{1}{2} a \times b$$

$$\delta A = \frac{\partial A}{\partial a} \cdot \delta a + \frac{\partial A}{\partial b} \cdot \delta b$$

$$\frac{\partial A}{\partial a} = \frac{b}{2} \quad \text{and} \quad \frac{\partial A}{\partial b} = \frac{a}{2}$$

$$\delta a = \pm \frac{1.5}{100} \cdot a \quad \delta b = \pm \frac{1.5}{100} \cdot b$$

$$\delta A = \frac{b}{2} \cdot \left(\pm \frac{1.5}{100} a \right) + \frac{a}{2} \cdot \left(\pm \frac{1.5}{100} b \right)$$

$$\delta A = \pm \frac{ab}{2} \left[\frac{3}{100} \right]$$

$$\delta A = \pm 3\% A$$

b) $h = \sqrt{a^2 + b^2} = (a^2 + b^2)^{1/2}$

$$\delta h = \frac{\partial h}{\partial a} \cdot \delta a + \frac{\partial h}{\partial b} \cdot \delta b$$

$$\frac{\partial h}{\partial a} = \frac{1}{2} (a^2 + b^2)^{-1/2} (2a) = \frac{a}{\sqrt{a^2 + b^2}}$$

$$\frac{\partial h}{\partial b} = \frac{1}{2} (a^2 + b^2)^{-1/2} (2b) = \frac{b}{\sqrt{a^2 + b^2}}$$

$$\delta a = \pm \frac{1.5 \cdot a}{100}$$

$$\delta b = \pm \frac{1.5 \cdot b}{100}$$

$$\delta h = \frac{a}{\sqrt{a^2 + b^2}} \left(\pm \frac{1.5 a}{100} \right) + \frac{b}{\sqrt{a^2 + b^2}} \left(\pm \frac{1.5 b}{100} \right)$$

$$\delta h = \pm \frac{a^2 + b^2}{\sqrt{a^2 + b^2}} \left(\frac{1.5}{100} \right)$$

$$= \pm \frac{1.5}{100} \sqrt{a^2 + b^2}$$

$$= \underline{\underline{0.015 \% h}}$$