

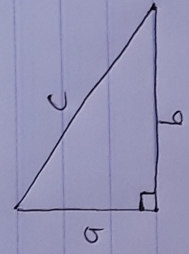
Mkpouto Ubong Obot
 Civil Engineering
 17/ENG03/015

ENG 281 (ENGINEERING MATHEMATICS)

The hypotenuse of a right angled triangle is denoted as c and the other two sides are denoted as a and b . If the possible error of measuring each of a and b is $\pm 1.5\%$, find the maximum possible error in calculating

- (a) The area of the triangle
- (b) The length of the hypotenuse

Solution



Area of triangle = $\frac{1}{2} ab$

Let the area $A = (a, b)$

$\Rightarrow \frac{\partial A}{\partial a} = \frac{b}{2}$

$\Rightarrow \frac{\partial A}{\partial b} = \frac{a}{2}$

$\delta A = \frac{\partial A}{\partial a} \cdot \delta a + \frac{\partial A}{\partial b} \cdot \delta b$

$= \frac{b}{2} \cdot \left(\pm \frac{1.5a}{100} \right) + \frac{a}{2} \cdot \left(\pm \frac{1.5b}{100} \right)$

$= \frac{ab}{2} \left(\pm \frac{1.5}{100} \right) + \frac{ab}{2} \left(\pm \frac{1.5}{100} \right)$

$= \pm \frac{ab}{2} \left(\frac{1.5}{100} \right) + \left(\pm \frac{ab}{2} \right) \left(\frac{1.5}{100} \right)$

$= \pm \frac{ab}{2} \left(\frac{1.5}{100} + \frac{1.5}{100} \right)$

$= \pm \frac{ab}{2} (0.015 + 0.015)$

$$= \pm 0.03 \frac{ab}{2}$$

$$\text{Recall } A = \frac{a+b}{2}$$

$$\therefore \delta A = \pm 0.03 A$$

$$(b) \quad C^2 = a^2 + b^2$$

$$\Rightarrow C = \sqrt{a^2 + b^2}$$

$$\text{Let } C = (a, b)^{-k}$$

$$\frac{\partial C}{\partial a}$$

$$= \frac{a}{\sqrt{a^2 + b^2}}$$

$$\frac{\partial C}{\partial b} = \frac{b}{\sqrt{a^2 + b^2}}$$

$$= \frac{b}{\sqrt{a^2 + b^2}}$$

$$\partial C = \frac{\partial C}{\partial a} \cdot \delta a + \frac{\partial C}{\partial b} \cdot \delta b$$

$$\Rightarrow \partial C = \frac{a}{\sqrt{a^2 + b^2}} \left(\pm \frac{1.59}{100} \right) + \frac{b}{\sqrt{a^2 + b^2}} \left(\pm \frac{1.5 \cdot b}{100} \right)$$

$$= \frac{1}{\sqrt{a^2 + b^2}} \left(\pm \frac{1.59a^2}{100} \right) + \frac{1}{\sqrt{a^2 + b^2}} \left(\pm \frac{1.5 \cdot b^2}{100} \right)$$

$$= \pm \frac{1}{\sqrt{a^2 + b^2}} \left(\frac{1.59a^2}{100} + \frac{1.5b^2}{100} \right)$$

$$= \pm \frac{1}{\sqrt{a^2 + b^2}} (0.0159a^2 + 0.015b^2)$$

$$= \pm \frac{1}{\sqrt{a^2 + b^2}} \cdot 0.015 (a^2 + b^2)$$

$$= \pm \frac{1}{C} \cdot 0.015 C^2 = \pm 0.015 C$$