

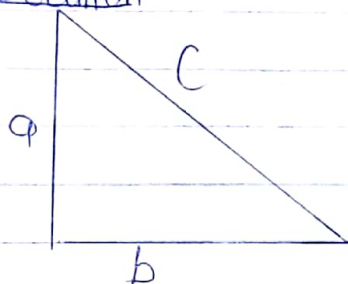
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Assignment

The hypotenuse of a right angled triangle is denoted as C , and the other two sides are denoted as a & b . If the possible error of measuring each of a and b is $\pm 1.5\%$, find the maximum possible error in calculating

- the area of the triangle
- the length of the hypotenuse

Solution



$$\text{Area of triangle} = \frac{1}{2} ab$$

$$\rightarrow A = \frac{1}{2} ab$$

$$\text{let } A = (a, b)$$

$$\rightarrow \frac{\delta A}{\delta a} = \frac{b}{2}$$

$$\frac{\delta A}{\delta b} = \frac{a}{2}$$

$$\delta C = \frac{a}{\sqrt{a^2 + b^2}} \left(\pm \frac{1.5a}{100} \right) + \frac{b}{\sqrt{a^2 + b^2}} \left(\pm \frac{1.5b}{100} \right)$$

$$= \frac{1}{\sqrt{a^2 + b^2}} \left(\pm \frac{1.5a^2}{100} \right) + \frac{1}{\sqrt{a^2 + b^2}} \left(\pm \frac{1.5b^2}{100} \right)$$

$$\hat{=} \pm \frac{1}{\sqrt{a^2 + b^2}} \left(\frac{1.5a^2}{100} + \frac{1.5b^2}{100} \right)$$

$$= \pm \frac{1}{\sqrt{a^2 + b^2}} (0.015a^2 + 0.015b^2)$$

$$= \pm \frac{1 \cdot 0.015(a^2 + b^2)}{\sqrt{a^2 + b^2}}$$

$$= \pm \frac{1 \cdot 0.015c^2}{c}$$

$$= \pm 0.015c$$

$$\delta A = \frac{\delta A}{\delta a} \cdot \delta a + \frac{\delta A}{\delta b} \cdot \delta b$$

$$= \frac{b}{2} \cdot \left(\pm \frac{1 \cdot 3a}{100} \right) + \frac{a}{2} \cdot \left(\pm \frac{1.5b}{100} \right)$$

$$= \frac{3b}{2} \left(\pm \frac{1.5}{100} \right) + \frac{ab}{2} \left(\pm \frac{1.5}{100} \right)$$

$$= \pm \frac{3b}{2} \left(\frac{1.5}{100} \right) + \left(\pm \frac{ab}{2} \right) \left(\frac{15}{100} \right)$$

$$= \pm \frac{3b}{2} (0.015 + 0.015)$$

$$= \pm 0.03 \frac{3ab}{2}$$

recall $A = \frac{ab}{2}$

$$\Rightarrow \delta A = \pm 0.03A$$

$$b) c^2 = a^2 + b^2$$
$$\Rightarrow c = \sqrt{a^2 + b^2}$$
$$= (a^2 + b^2)^{1/2}$$

$$\text{let } c = (a, b)$$

$$\frac{\partial c}{\partial a} = a(a^2 + b^2)^{-1/2}$$
$$= \frac{a}{\sqrt{a^2 + b^2}}$$

$$\frac{\partial c}{\partial a} = a(a^2 + b^2)^{-1/2}$$
$$= \frac{a}{\sqrt{a^2 + b^2}}$$

$$\frac{\partial c}{\partial b} = b(a^2 + b^2)^{-1/2}$$
$$= \frac{b}{\sqrt{a^2 + b^2}}$$

$$\frac{\partial c}{\partial a} = \frac{\partial c}{\partial a} \cdot da + \frac{\partial c}{\partial b} \cdot db$$