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DEPARTMENT: COMPUTER ENGINEERING

The hypotenuse of a right-angled triangle is denoted as c , and the other two sides are denoted as a and b . If the possible error of measuring each of a and b is $\pm 1.5\%$, find the maximum possible error in calculating:

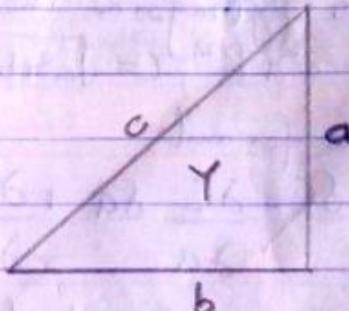
- (a) the area of the triangle, and
- (b) the length of the hypotenuse.

(a) The area of the triangle,

Let Y be the area of the triangle

Using the formula, $\text{Area} = \frac{1}{2} \times \text{base} \times \text{height}$

$$\text{Area}, Y = \frac{1}{2} \times b \times a = \frac{ab}{2}$$



Applying the formula

$$\delta Y = \frac{\partial Y}{\partial a} \cdot \delta a + \frac{\partial Y}{\partial b} \cdot \delta b$$

$$\frac{\partial Y}{\partial a} = \frac{b}{2}, \quad \frac{\partial Y}{\partial b} = \frac{a}{2}$$

$$\delta a = \pm \frac{1.5}{100} \text{ of } a = \pm \frac{1.5a}{100}, \quad \delta b = \pm \frac{1.5}{100} \text{ of } b = \pm \frac{1.5b}{100}$$

$$\delta Y = \frac{b}{2} \left(\pm \frac{1.5a}{100} \right) + \frac{a}{2} \left(\pm \frac{1.5b}{100} \right)$$

$$\delta Y = \frac{ab}{2} \left(\pm \frac{1.5}{100} \right) + \frac{ab}{2} \left(\pm \frac{1.5}{100} \right)$$

$$\delta Y = \frac{\pm ab}{2} \left(\frac{1.5}{100} \right) + \frac{\pm ab}{2} \left(\frac{1.5}{100} \right)$$

$$\delta Y = \frac{\pm ab}{2} \left(\frac{1.5 + 1.5}{100} \right)$$

$$\delta Y = \frac{\pm ab}{2} \left(\frac{3}{100} \right)$$

$$\text{Since } T = \frac{ab}{2}, \quad \delta T = \pm \star \left(\frac{3}{100} \right)$$

Therefore, the maximum possible error in calculating the area of the triangle,
 $\delta Y = 3$ per cent of the area of the triangle (3% of Y)

(b) the length of the hypotenuse.

From Pythagoras theorem,

$$c^2 = a^2 + b^2, \quad c = \sqrt{a^2 + b^2} \text{ or } (a^2 + b^2)^{1/2}$$

Using the formula,

$$\delta c = \frac{\partial c}{\partial a} \delta a + \frac{\partial c}{\partial b} \delta b$$

$\frac{\partial c}{\partial a}$, using the formula for function of a function $\left(\frac{\partial f}{\partial a} = \frac{\partial f}{\partial u} \times \frac{\partial u}{\partial a} \right)$

$$\text{Let } u = a^2 + b^2, \quad \frac{\partial u}{\partial a} = 2a, \quad \frac{\partial c}{\partial a} = \frac{1}{2} (u)^{-1/2} = \frac{1}{2} (a^2 + b^2)^{-1/2}$$

$$\frac{\partial c}{\partial a} = \frac{1}{2} (a^2 + b^2)^{-1/2} \times (2a) = \frac{2a}{2} (a^2 + b^2)^{-1/2} = a(a^2 + b^2)^{-1/2} \text{ or } \frac{a}{\sqrt{a^2 + b^2}}$$

$\frac{\partial c}{\partial b}$, using the formula for function of a function $\left(\frac{\partial f}{\partial b} = \frac{\partial f}{\partial u} \times \frac{\partial u}{\partial b} \right)$

$$\text{Let } u = a^2 + b^2, \quad \frac{\partial u}{\partial b} = 2b, \quad \frac{\partial c}{\partial b} = \frac{1}{2} (u)^{-1/2} = \frac{1}{2} (a^2 + b^2)^{-1/2}$$

$$\frac{\partial c}{\partial b} = \frac{1}{2} (a^2 + b^2)^{-1/2} \times (2b) = \frac{2b}{2} (a^2 + b^2)^{-1/2} = b(a^2 + b^2)^{-1/2} \text{ or } \frac{b}{\sqrt{a^2 + b^2}}$$

From question (a) we know that

$$\delta a = \frac{\pm 1.5a}{100} \quad \text{and} \quad \delta b = \frac{\pm 1.5b}{100}$$

$$\delta C = \frac{a}{\sqrt{a^2+b^2}} \left(\pm \frac{1.5a}{100} \right) + \frac{b}{\sqrt{a^2+b^2}} \left(\pm \frac{1.5b}{100} \right)$$

$$\delta C = \frac{\pm 1.5a^2}{(100)(\sqrt{a^2+b^2})} + \frac{\pm 1.5b^2}{100(\sqrt{a^2+b^2})}$$

$$\delta C = \frac{\pm 1.5(a^2+b^2)}{(100)(\sqrt{a^2+b^2})}$$

$$\delta C = \frac{\pm 1.5}{100} \left(\frac{1}{\sqrt{a^2+b^2}} \right)$$

Since the length of the hypotenuse, $c = \sqrt{a^2+b^2}$

$$\delta C = \frac{\pm 1.5}{100} (c)$$

Therefore, the maximum possible error in calculating the length of the hypotenuse, $\delta C = 1.5$ percent of the length of the hypotenuse (1.5% of c)