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ENG 231 [Engineering Mathematics].

The hypotenuse of a right-angled triangle is denoted as c, and the other two sides are denoted as a and b. If the possible error of measuring each of a and b is ± 1.5%, find the maximum possible error in calculating:
(a) the area of the triangle and
(b) the length of the hypotenuse.

Solution

(a) Area = ½ b·h
= ½ · b · a
= ba/2.

$\delta A = \frac{\partial A}{\partial a} \cdot \delta a + \frac{\partial A}{\partial b} \cdot \delta b$

$= \frac{b}{2} \cdot \frac{1.5}{100} a + \frac{a}{2} \cdot \frac{1.5}{100} \cdot b$

± 1.5% = [3/2 ÷ 100] b
= 3b/200

b/2 [± 3b/200] + a/2 [± 3a/200].
= ± a·b/2 [3/200 + 3/200].
= ± A [3/100]
ΔA = ± 3 %.

(b) $c = \sqrt{a^2+b^2}$
$= (a^2+b^2)^{\frac{1}{2}}$.
$dc/da = \frac{1}{2}(a^2+b^2)^{-1/2} \cdot 2a \left[\frac{dc}{du} \times \frac{du}{da}\right]$.
$= u[a^2+b^2]$
$dc/du = u^{-1/2}/2$

$\partial c/\partial a = 1/2 [a^2+b^2]^{-1/2} \cdot 2a$

$\partial c/\partial b = 1/2 [a^2+b^2]^{-1/2} \cdot 2b = b/\sqrt{a^2+b^2}$

$= \left[\frac{a^2+b^2}{2}\right]^{-1/2} \cdot 2a$

$\partial a = \pm 3a/200 \; ; \; \partial b = \pm 3b/200$

$\partial c = \partial c/\partial a \cdot \partial a + \partial c/\partial b \cdot \partial b$

$= \left[\frac{a}{\sqrt{a^2+b^2}} \pm \frac{3a}{200}\right] + \left[\frac{b}{\sqrt{a^2+b^2}} + \frac{3b}{200}\right]$

$= \frac{3}{200}\left[\frac{a^2+b^2}{\sqrt{a^2+b^2}}\right]$

$= \frac{3}{200}\left[\frac{c^2}{c}\right]$

$= 3/200 [c]$

$= 1.5\%$ of c.