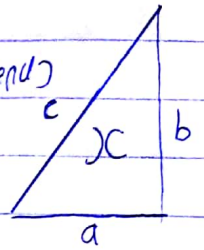
Modilim Tochukwu Adrian TAModilim
17/Eng 04/04/ Eng 235 assignment
Electrical Engineering

Question: The hypotenuse of a right angle triangle is denoted ~~ΔABC~~ as C and the other 2 sides are a & b. If possible error of measuring each side is ±1.5%, find the maximum possible error in calculating.

a) The area of triangle b) The length of hypotenuse



Solution

Let x = area of triangle $\Rightarrow$ Area = $\frac{1}{2} b \times h$

$\therefore b = a$, $h = b$

$\therefore x = \frac{1}{2} a \times b \Rightarrow ab/2$

Using formula $\delta x = \frac{\partial x}{\partial a} \cdot \delta a + \frac{\partial x}{\partial b} \cdot \delta b \Rightarrow 1$

$\frac{\partial x}{\partial a} = \frac{b}{2} \Rightarrow 2$, $\frac{\partial x}{\partial b} = \frac{a}{2} \Rightarrow 3$

$\therefore$ Substituting eqn 2 and 3 into eqn 1.

$$\delta x = \frac{b}{2}\left(\frac{\pm 1.5 a}{100}\right) + \frac{a}{2}\left(\frac{\pm 1.5 b}{100}\right)$$

$$\delta x = \pm\frac{ab}{2}\left(\frac{1.5}{100}\right) + \left(\pm\frac{ab}{2}\left(\frac{1.5}{100}\right)\right)$$

$$\delta x = \pm\frac{ab}{2}\left(\frac{1.5}{100} + \frac{1.5}{100}\right)$$

$$\delta x = \pm\frac{ab}{2}\left(\frac{3}{100}\right) - 4$$

$\therefore$ recall $x = ab/2$ - 5)

Substituting eq 5) into eqn 4

$$\delta x = \pm x\,\frac{3}{100}$$

$$\delta x = \pm 3\%\, x$$

$\therefore$ Therefore maximum possible error for Area (x) is ±3% (3 percent) of the area

b) The length of hypotenuse

Using pythagoras theorem

$C^2 = a^2 + b^2$

$C = \sqrt{a^2+b^2} \Rightarrow C = (a^2+b^2)^{1/2}$

Using formula $\delta C = \frac{\partial C}{\partial a}\delta a + \frac{\partial C}{\partial b}\delta b$ — 1)

$C = (a^2+b^2)^{1/2}$

let $V = a^2+b^2$, $\frac{\partial V}{\partial a} = 2a$

$C = V^{1/2}$

$\frac{\partial C}{\partial V} = \frac{1}{2}V^{-1/2} = \frac{1}{2V^{1/2}}$

$\frac{\partial C}{\partial a} = \frac{\partial V}{\partial a} \times \frac{\partial C}{\partial V} = 2a \times \frac{1}{2V^{1/2}} = \frac{a}{V^{1/2}} = \frac{a}{(a^2+b^2)^{1/2}}$ — 2)

To find $\partial C/\partial b$

$C = (a^2+b^2)^{1/2}$

let $Z = a^2+b^2$, $\partial Z/\partial b = 2b$

$C = Z^{1/2}$, $\partial C/\partial Z = 1/2 \times Z^{-1/2} = 1/2 Z^{1/2} = 1/2(a^2+b^2)^{1/2}$

$\frac{\partial C}{\partial b} = \frac{\partial C}{\partial Z} \times \frac{\partial Z}{\partial a} = \frac{1}{2(a^2+b^2)^{1/2}} \times 2b$

$\frac{\partial C}{\partial b} = \frac{b}{(a^2+b^2)^{1/2}}$ — 3)

$\therefore \delta C$ = Substitute eqn 2) & 3) into eqn 1)

$\Rightarrow \delta C = \frac{a}{(a^2+b^2)^{1/2}}\left(\frac{\pm 1.5a}{100}\right) + \frac{b}{(a^2+b^2)^{1/2}}\left(\frac{\pm 1.5b}{100}\right)$

$\delta C = \frac{a^2}{(a^2+b^2)^{1/2}}\left(\frac{\pm 1.5}{100}\right) + \frac{b^2}{(a^2+b^2)^{1/2}}\left(\frac{\pm 1.5}{100}\right)$

$\delta C = \frac{\pm 1.5}{(a^2+b^2)^{1/2}}(a^2+b^2)$

$\delta C = \frac{\pm 1.5}{(a^2+b^2)^{1/2}}((a^2+b^2)^{1/2}(a^2+b^2)^{1/2})$

$\delta C = \frac{\pm 1.5}{100}(a^2+b^2)^{1/2}$ $\quad | (a^2+b^2)^{1/2} = C$

$\therefore \delta C = \pm 1.5\%\ C$

The maximum possible error for hypotenuse is $\pm 1.5\%$ of C