SANI AHMAD DAHIRU
19/ENG03/047
CIVIL ENGINEERING

ENGINEERING MATHEMATICS

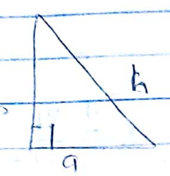
Assignment III

The hypotenuse of a right angled triangle is denoted as c, and the other two sides are denoted as a and b. If the possible error of measuring each of a and b is $\pm 1.5\%$. Find the maximum possible error in calculating:

a) Area of the triangle
b) length of the hypotenuse

Soln

Recall $A = \frac{1}{2}ab$



$$\delta A = \frac{\delta A}{\delta a} \cdot \delta a + \frac{\delta A}{\delta b} \cdot \delta b$$

$$\frac{\partial A}{\partial a} = \frac{1}{2} \cdot b = \frac{b}{2}, \quad \frac{\partial A}{\partial b} = \frac{1}{2} a = \frac{a}{2}$$

$$\delta a = \pm \frac{1.5}{100} a, \quad \delta b = \pm \frac{1.5}{100} b$$

$$= \pm \frac{3a}{200} + \delta b = \pm \frac{3b}{200}$$

$$\delta A = \frac{\partial A}{\partial a} \cdot \delta a + \frac{\partial A}{\partial b} \cdot \delta b$$

$$= \frac{b}{2} \times \pm \frac{3a}{200} + \frac{a}{2} \times \pm \frac{3b}{200}$$

$$= \frac{ab}{2} \pm \left[\frac{3}{200} + \frac{3}{200} \right] = \frac{6}{200} = \frac{3}{100}$$

$$= \frac{ab}{2} \pm \frac{3}{100} = 3\%$$

$\longrightarrow$ Recall $h = \sqrt{a^2+b^2} = (a^2+b^2)^{1/2}$

let $u = a^2+b^2$ $\quad h = u^{1/2}$ $\quad \frac{\partial h}{\partial u} = \frac{1}{2} u^{-1/2}$

$$\frac{\partial h}{\partial a} = \frac{\partial h}{\partial u} \times \frac{\partial u}{\partial a} = \frac{1}{2} u^{-1/2} \times 2a = a/\sqrt{a^2+b^2}$$

$$\frac{\partial h}{\partial b} = \frac{\partial h}{\partial u} \times \frac{\partial u}{\partial b} = \frac{1}{2} u^{-1/2} \times 2b = b/\sqrt{a^2+b^2}$$

$$\frac{\partial h}{\partial a} \cdot \delta a + \frac{\partial h}{\partial b} \cdot \delta b$$

$$\delta a = \pm \frac{3a}{200}, \quad \delta b = \pm 3b/200$$

$$\delta h = \frac{a}{\sqrt{a^2+b^2}} \times \pm \frac{3a}{200} + \frac{b}{\sqrt{a^2+b^2}} \times \pm \frac{3b}{200}$$

$$\delta h = \frac{a^2+b^2}{\sqrt{a^2+b^2}} \left[\frac{3}{200} \right] = 3/200 = 1.5\%$$