The hypotenuse of a right-angled triangle is denoted as c, and the other two sides are denoted as a and b. If the possible error of measuring each of a and b is $\pm 1.5\%$, find the maximum possible error in calculating:

(a) the area of the triangle and

(b) the length of the hypotenuse.

Solution

(a) $Area = \frac{1}{2} b \cdot h$

$= \frac{1}{2} \cdot b \cdot a$

$= ba/2$.

$\delta A = \frac{\partial A}{\partial a} \cdot \delta a + \frac{\partial A}{\partial b} \cdot \delta b$

$= \frac{b}{2} \cdot \frac{1.5}{100} a + \frac{a}{2} \cdot \frac{1.5}{100} \cdot b$

$\pm 1.5\% = [3/2 \div 100] b$

$= 3b/200$

$b/2 [\pm 3b/200] + a/2 [\pm 3a/200]$.

$= \pm a \cdot b/2 \; [3/200 + 3/200]$.

$= \pm A [3/100]$

$\Delta A = \pm 3\%$.

(b) $C = \sqrt{a^2+b^2}$

$= (a^2+b^2)^{\frac{1}{2}}$.

$dc/\partial a = \frac{1}{2}(a^2+b^2)^{-1/2} \cdot 2a \left[\frac{dc}{du} \times \frac{du}{da} \right]$.

$= u[a^2+b^2]$

$dc/du = u^{-1/2}/2$

$\partial c/\partial a = \frac{1}{2}[a^2+b^2]^{-1/2} \cdot 2a.$

$\partial c/\partial b = \frac{1}{2}[a^2+b^2]^{-1/2} \cdot 2b = b/\sqrt{a^2+b^2}$

$= \frac{[a^2+b^2]^{-1/2}}{2} \cdot 2a$

$\partial a/a = \partial b/b$ $\quad \partial a = +3a/200$; $\partial b = \pm 3b/200$

$\partial c = \partial c/\partial a \cdot \partial a + \partial c/\partial b \cdot \partial b.$

$= \left[\frac{a}{\sqrt{a^2+b^2}} \pm \frac{3a}{200}\right] + \left[\frac{b}{\sqrt{a^2+b^2}} + \frac{3b}{200}\right]$

$= \frac{3}{200}\left[\frac{a^2+b^2}{\sqrt{a^2+b^2}}\right]$

$= \frac{3}{200}\left[\frac{c^2}{c}\right]$

$= 3/200[c]$

$= 1.5\%$ of C.