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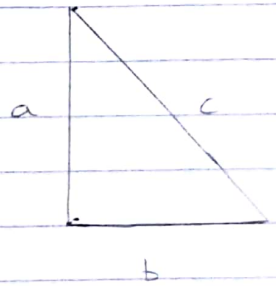
17/ENG03/020
CIVIL ENGINEERING
ENG 281

QUESTION

The hypotenuse of a right-angle triangle is denoted as c, and the other two sides are denoted as a and b. If the possible error of measuring each of a and b is ± 1.5 find the maximum possible error in calculating

a) The area of the triangle, and

b) The length of the hypotenuse



Recall, $A = \frac{1}{2} ba$

$h = \sqrt{a^2 + b^2}$

$$\Rightarrow \delta A = \frac{\partial A}{\partial a} \cdot \delta a + \frac{\partial A}{\partial b} \cdot \delta b$$

$$= \frac{b}{2} \cdot \delta a + \frac{a}{2} \cdot \delta b$$

$$\delta b = \pm \frac{1.5}{100} \times b \qquad \delta a = \pm \frac{1.5}{100} \times a$$

$$= \pm \frac{3b}{200} \qquad = \pm \frac{3a}{200}$$

$$= \frac{b}{2} \cdot \pm \frac{3b}{200} + \frac{a}{2} \pm \frac{3a}{200}$$

$$= \frac{3ab}{2} \left(\pm \frac{1}{200} + \pm \frac{1}{200} \right)$$

$$= \frac{3ab}{2} \left(\pm \frac{1}{100} \right)$$

$$\text{Area} = \frac{ab}{2}$$

$= 3A \left(\pm \frac{1}{100} \right)$

$= A \left(\pm \frac{3}{100} \right)$

$= A$ increases / decreases by 3%

$\Rightarrow \delta h = \frac{\partial h}{\partial a} \cdot \delta a + \frac{\partial h}{\partial b} \cdot \delta b$

$h = (a^2 + b^2)^{\frac{1}{2}}$

$u = a^2 + b^2$

$h = u^{\frac{1}{2}}$

$\frac{\partial h}{\partial a} = \frac{\partial h}{\partial u} \times \frac{\partial u}{\partial a}$

$= \frac{1}{2} u^{-\frac{1}{2}} \times 2a$

$= \frac{a}{\sqrt{a^2+b^2}}$

$\frac{\partial h}{\partial a} = \frac{\partial h}{\partial u} \times \frac{\partial u}{\partial b}$

$= \frac{1}{2} u^{-\frac{1}{2}} \cdot 2b$

$= \frac{b}{\sqrt{a^2+b^2}}$

$= \frac{a}{\sqrt{a^2+b^2}} \cdot \pm \frac{3a}{200} + \frac{b}{\sqrt{a^2+b^2}} \pm \frac{3b}{200}$

$= \frac{a^2}{\sqrt{a^2+b^2}} \cdot \pm \frac{3}{200} + \frac{b^2}{\sqrt{a^2+b^2}} \cdot \pm \frac{3}{200}$

$= \frac{a^2+b^2}{\sqrt{a^2+b^2}} \left(\pm \frac{3}{100} \right)$

$h = \frac{a^2+b^2}{\sqrt{a^2+b^2}}$

$= h$ increase / decreases by 3%