

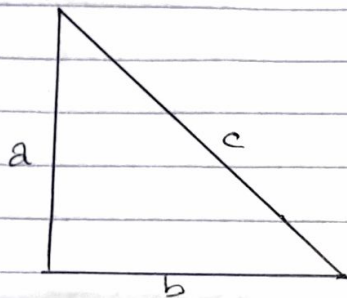
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MATHEMATICS (ENG 281)

QUESTION

The hypotenuse of a right-angle triangle is denoted as c , and the other two sides are denoted as a and b . If the possible error of measuring each of a and b is $\pm 1.5\%$ find the maximum possible error in calculating

- the area of the triangle, and
- the length of the hypotenuse



Recall, $A = \frac{1}{2}ab$
 $c = \sqrt{a^2 + b^2}$

$$\Rightarrow \delta A = \frac{\delta A}{A} \cdot \left(\frac{\delta a}{a} + \frac{\delta b}{b} \right) \cdot A$$

$$= \frac{b}{2} \cdot \frac{\delta a}{a} + \frac{a}{2} \cdot \frac{\delta b}{b}$$

$$\delta b = \pm \frac{1.5}{100} \times b$$

$$\delta a = \pm \frac{1.5}{100} \times a$$

$$= \pm \frac{3 \cdot b}{200}$$

$$= \pm \frac{3a}{200}$$

$$= \frac{b}{2} \cdot \frac{3b}{200} + \frac{a}{2} \cdot \frac{3a}{200}$$

$$= \frac{3ab}{2} \left(\frac{1}{200} + \frac{1}{200} \right)$$

$$= \frac{3ab}{2} \left(\frac{1}{100} \right)$$

$$\text{Area} = \frac{ab}{2}$$

$$= 3A \left(\frac{1}{100} \right)$$

$$= A \left(\frac{3}{100} \right)$$

= h increases/decreases by 3%

$$\rightarrow \delta h = \frac{\partial h}{\partial a} \cdot \delta a + \frac{\partial h}{\partial b} \cdot \delta b$$

$$h = (a^2 + b^2)^{1/2}$$

$$h^2 = a^2 + b^2$$

$$h = u^{1/2}$$

$$\frac{\partial h}{\partial a} = \frac{\partial h}{\partial u} \times \frac{\partial u}{\partial a}$$

$$= \frac{1}{2} u^{-1/2} \times 2a$$

$$= \frac{a}{\sqrt{a^2 + b^2}}$$

$$\frac{\partial h}{\partial a} = \frac{\partial h}{\partial u} \times \frac{\partial u}{\partial a}$$

$$= \frac{1}{2} u^{-1/2} \cdot 2b$$

$$= \frac{b}{\sqrt{a^2 + b^2}}$$

$$= \frac{a}{\sqrt{a^2 + b^2}} \cdot \frac{3a}{200} + \frac{b}{\sqrt{a^2 + b^2}} \cdot \frac{3b}{200}$$

$$= \frac{a^2}{\sqrt{a^2 + b^2}} \cdot \frac{3}{200} + \frac{b^2}{\sqrt{a^2 + b^2}} \cdot \frac{3}{200}$$

$$= \frac{a^2 + b^2}{\sqrt{a^2 + b^2}} \left(\frac{3}{200} \right)$$

$$h = \frac{a^2 + b^2}{\sqrt{a^2 + b^2}}$$

= h increase/decreases by 3%