METU CHIAGOGIEM
MECHATRONICS
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ENG 281

The hypotenuse of a right angle triangle is denoted as c, and the other two sides are denoted as a and b. If the possible error of measuring each of a and b is $\pm$ 1.5%. Find the maximum possible error in calculating:

(a) the area of the triangle
(b) the length of the hypotenuse

Recall $A = \frac{1}{2}ba$

$h = \sqrt{a^2 + b^2}$

$\Rightarrow \delta A = \frac{\partial A}{\partial a} \cdot \delta a + \frac{\partial A}{\partial b} \cdot \delta b$

$= \frac{b}{2} \cdot \delta a + \frac{a}{2} \cdot \delta b$

$\delta b = \frac{-1.5}{100} \times b$ $\qquad \delta a = \frac{\pm 1.5}{100} \times a$

$= \frac{\pm 3 \cdot b}{200}$ $\qquad = \frac{\pm 3 \cdot a}{200}$

$= \frac{b}{2} \cdot \frac{\pm 3b}{200} + \frac{a}{2} \cdot \frac{\pm 3a}{200}$

$= \frac{3ab}{2}\left(\frac{\pm 1}{200} + \frac{\pm 1}{200}\right)$

$= \frac{3ab}{2}\left(\frac{\pm 1}{100}\right)$

$Area = \frac{ab}{2}$

$\delta A = \left(\frac{\pm 1}{100}\right)$

$A = \left(\frac{\pm 3}{100}\right)$ $\therefore$ A increases/decreases by 3%

$$\delta h = \frac{\partial h}{\partial a} \cdot \delta a + \frac{\partial h}{\partial b} \cdot \delta b$$

$$h = (a^2 + b^2)^{1/2}$$

$$u = a^2 + b^2$$

$$h = u^{1/2}$$

$$\frac{\partial h}{\partial a} = \frac{\partial h}{\partial u} \times \frac{\partial u}{\partial a}$$

$$= \tfrac{1}{2} u^{-1/2} \times 2a$$

$$= \frac{a}{\sqrt{a^2+b^2}}$$

$$= \left(\frac{a}{\sqrt{a^2+b^2}} \pm \frac{3a}{200}\right) + \left(\frac{b}{\sqrt{a^2+b^2}} \pm \frac{3b}{200}\right)$$

$$= \frac{a^2}{\sqrt{a^2+b^2}} \pm \frac{3}{200} + \frac{b^2}{\sqrt{a^2+b^2}} \pm \frac{3}{200}$$

$$= \frac{a^2+b^2}{\sqrt{a^2+b^2}} \left(\pm \frac{3}{200}\right)$$

$$h = \frac{a^2+b^2}{\sqrt{a^2+b^2}}$$

$\therefore h$ increases / decreases by 3%