The hypotenuse of a right-angled triangle is denoted as c, and the other two sides are denoted as a and b. If the possible error of measuring each of a and b is ±1.5%. Find the maximum possible error in calculating:

a) the area of the triangle, and

b) the length of the hypotenuse.

$A = \frac{1}{2} ab$

a) $\delta A = \frac{\partial A}{\partial a} \cdot \delta a + \frac{\partial A}{\partial b} \cdot \delta b$

$= \frac{b}{2} \cdot \delta a + \frac{a}{2} \cdot \delta b$

$= \frac{b}{2}\left(\pm \frac{1.5}{100} a\right) \pm \frac{a}{2}\left(\pm \frac{1.5}{100} b\right)$

$= \pm \frac{ab}{2}(0.015) \pm \frac{ab}{2}(0.015)$

$\delta A = \pm A(0.03)$

$\delta A = \pm 3\% A$

b) Given the given triangle is of a right-angled -

$c = \sqrt{a^2 + b^2}$

$c^2 = a^2 + b^2$

$\delta c = \frac{\partial c}{\partial a} \cdot \delta a + \frac{\partial c}{\partial b} \cdot \delta b$

For $\frac{\partial c}{\partial a}$

$c = (a^2+b^2)^{1/2}$

let $a^2+b^2 = v$

$c = v^{1/2}$

$\frac{dc}{dv} = \frac{v^{-1/2}}{2}$

$v = a^2+b^2$

$\frac{dv}{da} = 2a$

$\therefore \frac{\partial c}{\partial a} = \frac{dc}{dv} \times \frac{dv}{da}$

$= \frac{v^{-1/2}}{2} \cdot 2a$

$\frac{\partial c}{\partial a} = a(a^2+b^2)^{-1/2}$

For $\frac{\partial c}{\partial b}$

$c = (a^2+b^2)^{1/2}$

let $a^2+b^2 = v$

$c = v^{1/2}$

$\frac{dc}{dv} = \frac{v^{-1/2}}{2}$

$v = a^2+b^2$

$\frac{dv}{db} = 2a$

$\therefore \frac{\partial c}{\partial b} = \frac{dc}{dv} \times \frac{dv}{db}$

$= \frac{v^{-1/2}}{2} \cdot 2a$

$\frac{\partial c}{\partial b} = b(a^2+b^2)^{-1/2}$

$\delta c = a(a^2+b^2)^{-1/2} + (\pm 0.015a) + b(a^2+b^2)^{-1/2} + (\pm 0.015b)$

$= a^2(a^2+b^2)^{-1/2} + (\pm 0.015) + b^2(a^2+b^2)^{-1/2} + (\pm 0.015)$

Recall that $(a^2+b^2)^{-1/2} = c$

$= \frac{a^2}{c}(\pm 0.015) + \frac{b^2}{c}(\pm 0.015)$

$= \pm 0.015\left[\frac{a^2}{c} + \frac{b^2}{c}\right]$

$= \pm 0.015\left[\frac{1}{c}(a^2+b^2)\right]$

Recall $a^2+b^2 = c$

$= \pm 0.015\left[\frac{1}{c} \cdot c^2\right]$

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$\delta c = \pm(0.015)c$

$\delta c = \pm 1.5\% \, c$