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Civil Engineering
17/ENG03/052

ENG 281 (Engineering Mathematics)

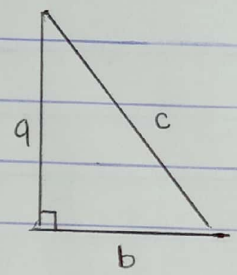
Question:

The hypotenuse of a right angled triangle is denoted as c and the other two sides are denoted as a and b. If the possible error of measuring each of a and b is $\pm 1.5\%$, find the maximum possible error in calculating

a) The area of the triangle.
b) The length of the hypotenuse.

Solution

a)



Area of triangle $= \frac{1}{2}ab$

let the area $A = (a, b)$

$$\Rightarrow \frac{\partial A}{\partial a} = \frac{b}{2}$$

$$\Rightarrow \frac{\partial A}{\partial b} = \frac{a}{2}$$

$$\delta A = \frac{\partial A}{\partial a} \cdot \delta a + \frac{\partial A}{\partial b} \cdot \delta b$$

$$= \frac{b}{2} \cdot \left(\frac{\pm 1.5a}{100}\right) + \frac{a}{2} \cdot \left(\frac{\pm 1.5b}{100}\right)$$

$$= \frac{ab}{2}\left(\frac{\pm 1.5}{100}\right) + \frac{ab}{2}\left(\frac{\pm 1.5}{100}\right)$$

$$= \pm\frac{ab}{2}\left(\frac{1.5}{100}\right) + \pm\frac{ab}{2}\left(\frac{1.5}{100}\right)$$

$$= \pm \frac{ab}{2}\left(\frac{1.5}{100} + \frac{1.5}{100}\right)$$

$$= \pm \frac{ab}{2}(0.015 + 0.015)$$

$$= \pm \frac{ab}{2}(0.03)$$

$$= \pm 0.03 \frac{ab}{2}$$

result $A = ab/2$

$\therefore \delta A = \pm 0.03 A$

b) $c^2 = a^2 + b^2$

$c = \sqrt{a^2 + b^2}$

Let $c = (a, b)$

$$\frac{\partial c}{\partial a} = a(a^2+b^2)^{-1/2}$$

$$= \frac{a}{\sqrt{a^2+b^2}}$$

$$\frac{\partial c}{\partial b} = b(a^2+b^2)^{-1/2}$$

$$= \frac{b}{\sqrt{a^2+b^2}}$$

$$\delta c = \frac{\partial c}{\partial a} \cdot \delta a + \frac{\partial c}{\partial b} \cdot \delta b$$

$$\delta c = \frac{a}{\sqrt{a^2+b^2}}\left(\pm \frac{1.5a}{100}\right) + \frac{b}{\sqrt{a^2+b^2}}\left(\pm \frac{1.5b}{100}\right)$$

$$= \frac{1}{\sqrt{a^2+b^2}}\left(\pm \frac{1.5a^2}{100}\right) + \frac{1}{\sqrt{a^2+b^2}}\left(\pm \frac{1.5b^2}{100}\right)$$

$$= \pm \frac{1}{\sqrt{a^2+b^2}}\left(\frac{1.5a^2}{100} + \frac{1.5b^2}{100}\right)$$

$$= \pm \frac{1}{\sqrt{a^2+b^2}} \left(0.015a^2 + 0.015b^2\right)$$

$$= \pm \frac{1}{\sqrt{a^2+b^2}} \cdot 0.015\left(a^2+b^2\right)$$

$$= \pm \frac{1}{c} \cdot 0.015c^2$$

$$= \pm 0.015 \# c$$