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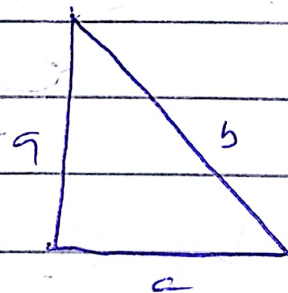
CIVIL ENGINEERING

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File 281

The hypotenuse of a right angle triangle is denoted as c and the other two sides are denoted as a and b . If the possible error of measuring each of a and b is $\pm 1.5\%$. Find the maximum possible error in calculating:

- (i) the area of the triangle
- (ii) the length of the hypotenuse



Recall $A = \frac{1}{2}bs$

$$h = \sqrt{a^2 + b^2}$$

$$\rightarrow \delta A = \frac{\partial A}{\partial a} \cdot \delta a + \frac{\partial A}{\partial b} \cdot \delta b$$

$$= \frac{b}{2} \cdot \delta a + \frac{a}{2} \cdot \delta b$$

$$\delta b = \frac{-1.5}{100} ab$$

$$\delta a = \frac{+1.5}{100} a^2$$

$$= \frac{+3.6}{200}$$

$$= \frac{+3.7}{200}$$

$$= \frac{b}{2} \cdot \frac{+3b}{200} + \frac{a}{2} \cdot \frac{+3a}{200}$$

$$= \frac{3ab}{2} \left(\frac{+1}{200} \times \frac{+1}{200} \right)$$

$$= \frac{3ab}{2} \left(\frac{+1}{100} \right)$$

$$\text{Area} = \frac{ab}{2}$$

$$\delta A = \left(\frac{+1}{100} \right)$$

$$A = \left(\frac{+3}{100} \right)$$

\therefore An increase/decrease by 3%

$$\Rightarrow \delta h = \frac{\partial h}{\partial a} \cdot \delta a + \frac{\partial h}{\partial b} \cdot \delta b$$

$$u = (a^2 + b^2)^{1/2}$$

$$u = a^2 + b^2$$

$$h = u^{1/2}$$

$$\frac{\partial h}{\partial a} = \frac{\partial h}{\partial u} \times \frac{\partial u}{\partial a}$$

$$= \frac{1}{2} u^{-1/2} \times 2a$$

$$= \frac{a}{\sqrt{a^2 + b^2}}$$

$$= \left(\frac{a}{\sqrt{a^2 + b^2}} + \frac{3a}{2m} \right) + \left(\frac{b}{\sqrt{a^2 + b^2}} + \frac{3b}{2m} \right)$$

$$= \frac{a^2}{\sqrt{a^2 + b^2}} + \frac{3}{2m} + \frac{b^2}{\sqrt{a^2 + b^2}} + \frac{3}{2m}$$

$$= \frac{a^2 + b^2}{\sqrt{a^2 + b^2}} \left(+ \frac{3}{m} \right)$$

$$h = \frac{a^2 + b^2}{\sqrt{a^2 + b^2}}$$

$\therefore h$ increases (decreases) by 3%