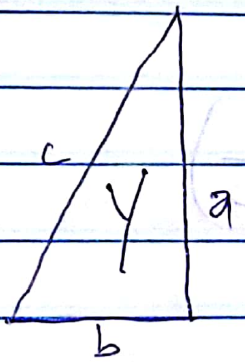
CHINEDUM PRUDENCE ESE
CHEMICAL ENGINEERING
17/ENG01/007
ENG 281

The hypotenus of right angled triangle is denoted as c and the other two sides are denoted as an a & b. If the possible error of measuring each of a & b is ±1.5%, find the maximum possible error in calculating:

a) The area of the triangle, and
b) The length of the hypotenus.



a) The area of triangle

let Y be the area of the triangle

Using the formula, Area = $\frac{1}{2}$ x base x height

Area, $Y = \frac{1}{2} \times b \times a = \frac{ab}{2}$

$$dy = \frac{dy}{da} \cdot da + \frac{dy}{db} \cdot db$$

$\frac{dy}{da} = \frac{b}{a}$, $\frac{dy}{db} = \frac{a}{2}$

$da = \pm\frac{1.5}{100}$ of $a = \pm\frac{1.5a}{100}$, $db = \pm\frac{1.5}{100}$ of $b = \pm\frac{1.5b}{100}$

$dy = \frac{b}{2}\left(\frac{\pm 1.5a}{100}\right) + \frac{a}{2}\left(\frac{\pm 1.5b}{100}\right)$

$dy = \frac{ab}{2}\left(\frac{\pm 1.5}{100}\right) + \frac{ab}{x}\left(\frac{\pm 1.5}{100}\right)$

$dy = \pm\frac{ab}{2}\left(\frac{1.5}{100}\right) + \pm\frac{ab}{2}\left(\frac{1.5}{100}\right)$

$dy = \pm\frac{ab}{2}\left(\frac{1.5+1.5}{100}\right)$, $dy = +\frac{ab}{2}\left(\frac{3}{100}\right)$

Since $Y = \frac{ab}{2}$, $dY = \pm A\left(\frac{3}{100}\right)$

Therefore, the maximum possible error in calculating the area of the triangle, dY = 3% of the area of the triangle (3% of Y)

b) The length of the hypotenuse

from Pythagoras,

$c^2 = a^2 + b^2$

using the formula

$$dc = \frac{dc}{da}da + \frac{dc}{db}db$$

$$\frac{dc}{da} = \frac{1}{2}(a^2+b^2)^{-1/2} \times (2a) = \frac{(2a)(a^2+b^2)^{-1/2}}{2} = a(a^2+b^2)^{-1/2} \text{ or } \frac{a}{\sqrt{a^2+b^2}}$$

$\frac{dc}{db}$ using the formula for function of a function of $\left(\frac{dc}{db} = \frac{dc}{du} \times \frac{du}{db}\right)$

Let $u = a^2+b^2$, $\frac{du}{db} = 2b$, $\frac{dc}{du} = \frac{1}{2}(u)^{-1/2} = \frac{1}{2}(a^2+b^2)^{-1/2}$

$$\frac{dc}{db} = \frac{1}{2}(a^2+b^2)^{-1/2} \times (2b) = \frac{2b(a^2+b^2)^{-1/2}}{2} = b(a^2+b^2)^{-1/2} \text{ or } \frac{b}{\sqrt{a^2+b^2}}$$

From question (a) we know that

$$\delta a = \pm\frac{1.5a}{100} \text{ and } \delta b = \pm\frac{1.5b}{100}$$

$$dc = \frac{a}{\sqrt{a^2+b^2}}\left(\frac{\pm 1.5a}{100}\right) + \frac{b}{\sqrt{a^2+b^2}}\left(\frac{\pm 1.5b}{100}\right)$$

$$dc = \frac{\pm 1.5a^2}{(100)(\sqrt{a^2+b^2})} + \frac{\pm 1.5b^2}{(100)(\sqrt{a^2+b^2})}$$

$$\delta c = \frac{\pm 1.5(a^2+b^2)}{(100)(\sqrt{a^2+b^2})}$$

$$\delta c = \frac{\pm 1.5}{100}\left(\sqrt{a^2+b^2}\right)$$

Since the length of hypotenus, $c = \sqrt{a^2+b^2}$

$$\delta c = \frac{\pm 1.5}{100}(c)$$

∴ Therefore, the maximum error in calculating the length of the hypotenus is (1.5% of c)