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The hypotenuse of a right-angled triangle is denoted as C and the other two sides are denoted as a and b. If the possible error of measuring each of a and b is $\pm 1.5\%$. Find the maximum possible error in calculating

a. The Area of the triangle
b. The length of the hypotenuse.

Soln

a. Let Z be the area of the triangle

$$Z = \frac{1}{2}ab$$

Applying the formula

$$\delta Z = \frac{\delta z}{\delta a} \cdot \delta a + \frac{\delta z}{\delta b} \cdot \delta b$$

$$\frac{\delta z}{\delta a} = \frac{b}{2}, \quad \frac{\delta z}{\delta b} = \frac{a}{2}$$

$$\delta a = \frac{\pm 1.5}{100} \text{ of } a = \frac{\pm 1.5a}{100}, \quad \delta b = \frac{\pm 1.5}{100} \log b = \frac{\pm 1.5b}{100}$$

$$\delta z = \frac{b}{2}\left(\frac{\pm 1.5a}{100}\right) + \frac{a}{2}\left(\frac{\pm 1.5b}{100}\right)$$

$$\delta z = \frac{ab}{2}\left(\frac{\pm 1.5a}{100}\right) + \frac{ab}{2}\left(\frac{\pm 1.5}{100}\right)$$

$$\delta z = \cancel{\frac{ab}{2}} \pm \frac{ab}{2}\left(\frac{1.5}{100}\right) + \pm \frac{ab}{2}\left(\frac{1.5}{100}\right)$$

$$\delta z = \frac{\pm ab}{2}\left(\frac{1.5 + 1.5}{100}\right)$$

$$\delta z = \frac{\pm ab}{2}\left(\frac{3}{100}\right)$$

Since $Z = \frac{ab}{2}$, $\delta z = \pm Z\left(\frac{3}{100}\right)$

Therefore the maximum possible error in calculating the area of the triangle $\delta z = 3$ percent of the area of the triangle (3% of z)

b. Length of c
from pythagoras
$c^2 = a^2+b^2 \Rightarrow c = \sqrt{a^2+b^2}$ or $(a^2+b^2)^{\frac{1}{2}}$

Using the

$$\delta c = \frac{\delta c}{\delta a} \cdot \delta a + \frac{\delta c}{\delta b} \cdot \delta b$$

$$\frac{\delta c}{\delta a} = \left(\frac{\delta c}{\delta u} \times \frac{\delta u}{\delta a}\right)$$

let $u = a^2+b^2$, $\frac{\delta u}{\delta a} = 2a$, $\frac{\delta c}{\delta u} = \frac{1}{2}(u)^{-\frac{1}{2}} = \frac{1}{2}(a^2+b^2)^{-\frac{1}{2}}$

$$\frac{\delta c}{\delta a} = \frac{1}{2}(a^2+b^2)^{-\frac{1}{2}} \times (2a) = \frac{2a(a^2+b^2)^{-\frac{1}{2}}}{2} = a(a^2+b^2)^{-\frac{1}{2}}$$

$$\frac{\delta c}{\delta b} = \left(\frac{\delta c}{\delta u} \times \frac{\delta u}{\delta b}\right) \quad \frac{\delta u}{\delta b} = 2b \quad \frac{\delta c}{\delta u} = \frac{1}{2}(u)^{-\frac{1}{2}}$$

$$= \frac{1}{2}(a^2+b^2)^{-\frac{1}{2}}$$

$$\frac{\delta c}{\delta b} = \frac{1}{2}(a^2+b^2)^{-\frac{1}{2}} \times (2b) = \frac{2b(a^2+b^2)^{-\frac{1}{2}}}{2} = b(a^2+b^2)^{-\frac{1}{2}}$$

From Area of Triangle,

$\delta a = \frac{\pm 1.5a}{100}$ and $\delta b = \frac{\pm 1.5b}{100}$

$$\delta c = \frac{a}{\sqrt{a^2+b^2}}\left(\frac{\pm 1.5a}{100}\right) + \frac{b}{\sqrt{a^2+b^2}}\left(\frac{\pm 1.5b}{100}\right)$$

$$\delta c = \frac{\pm 1.5a^2}{100(\sqrt{a^2+b^2})} + \frac{\pm 1.5b^2}{100(\sqrt{a^2+b^2})}$$

$$\delta c = \frac{\pm 1.5(a^2+b^2)}{(100)(\sqrt{a^2+b^2})}$$

$$c = \sqrt{a^2+b^2}$$

$$Sc = \frac{\pm 1.5}{100} \cdot (c)$$

Therefore, the maximum possible error in Calculating the length of the hypotenuse, Sc = 1.5 percent of the length of the hypotenuse (1.5% of C).