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16/ENG01/014
Chemical Engineering

$y = e^{x+x^2}$

Using the chain rule / Function of a function.

$y = e^u$ $\quad y = e^u$

$u = x + x^2$ $\quad dy/du = e^u$

$\frac{du}{dx} = 1 + 2x$

$dy/du \times du/dx = (2x+1)e^u$

$dy/dx = (2x+1)e^{(x+x^2)}$

$\therefore y' = (2x+1)(e^{x+x^2})$

$y'' \left(\frac{d^2y}{dx^2}\right) = [2x+1][(2x+1)e^{x^2+x}] + 2[e^{x^2+x}]$.

$y'' = (2x+1)((2x+1)e^{x^2+x}) + 2[e^{x^2+x}]$

where $e^{x^2+x} = y$

and $(2x+1)(e^{x^2+x}) = y'$

$\therefore y'' = y'(2x+1) + 2(y)$ proven

b) $y'' = y'(2x+1) + 2y$

$y'' - y'(2x+1) - 2y = 0$

Using Leibnitz theorem.

$u_1^n = y''$ $\quad v_1 = 1$ $\quad u^0 = y''$

$v_1' = 0$ $\quad u^n = y^{n+2}$

$W_1^n = u^n v + n u^{n-1} v' + \frac{n(n-1)}{2!} u^{n-2} v'' + \cdots$

$W_1^n = y^{n+2}(1) + n y^{n+1}(0) + \cdots$

$\therefore W_1^n = y^{n+2}$

Expansing $-y'(2x+1)$

$W_2^n = -2xy' \qquad W_3 = -y'$

$W_2^n = -2xy'$

$v^{(0)} = -2x \quad u = y'$

$v' = -2 \quad u' = y''$

$v'' = 0 \quad u^n = y^{n+1}$

$W_2^n = u^n v + n u^{n-1} v' + \frac{n(n-1)}{2!} u^{n-2} v'' + \cdots$

$W_2^n = y^{n+1}(-2x) + n y^n(-2) + \frac{n(n-1)}{2!} \times y^{n-1}(0)$

$W_2^n = -2x y^{n+1} - 2n y^n$

$W_3^n = -y' \quad u^0 = y' \quad v = -1$

$u' = y'' \quad v' = 0$

$u^n = y^{n+1}$

$W_3^n = y^{n+1}(-1) + 0 = -y^{n+1}$

$W_4^n = -2y \quad u = y \quad v = -2$

$u' = y' \quad v' = 0$

$u^n = y^n$

$W_4^n = -2y^n$

Adding all results

$W_1^n + W_2^n + W_3^n + W_4^n = y^{n+2} - 2x y^{n+1} - 2n y^n - (y^{n+1}) - 2y^n = 0$

$y^{n+2} - 2x y^{n+1} - 2n y^n - y^{n+1} - 2y^n = 0$

$y^{n+2} = 2x y^{n+1} + 2n y^n + y^{n+1} + 2y^n.$

$y^{n+2} = (2x+1) y^{n+1} + 2(n+1) y^n$ Proved

$y = x^3 e^{4x}$ $\rightarrow$ determine $y^{(5)}$

$u = e^{4x}$ $\quad v = x^3$

$u' = 4e^{4x}$ $\quad v' = 3x^2$

$u'' = 16e^{4x}$ $\quad v'' = 6x$

$u^n = 4^n e^{4x}$ $\quad v''' = 6$

$v^{IV} = 0$

$$y^n = u^n v + n u^{n-1} v' + \frac{n(n-1)}{2!} u^{n-2} v^2 + \frac{n(n-1)(n-2)}{3!} u^{n-3} v^3 \ldots$$

$$y^n = 4^n e^{4x} x^3 + n 4^{n-1} e^{4x} 3x^2 + \frac{n(n-1)}{2!} 4^{n-2} e^{4x} 6x + \frac{n(n-1)(n-2)}{6} \times 4^{n-3} e^{4x} 6.$$

$$y^n = 4^n e^{4x} x^3 + n 4^{n-1} e^{4x} 3x^2 + n(n-1) 4^{n-2} 3x + n(n-1)(n-2) 4^{n-3} e^{4x} \ldots$$

$$y^n = 4^{n-3} e^{4x} [4^3 x^3 + n 4^2 \times 3x^2 + n(n-1) 4 \times 3x + n(n-1)(n-2)]$$

$$y^n = 4^{n-3} e^{4x} [64x^3 + n 48x^2 + n(n-1) 12x + n(n-1)(n-2)]$$

$$y^5 = 4^{5-3} e^{4x} [64x^3 + 5(48)x^2 + 5(4) 12x + 5(4)(3)]$$

$$y^5 = 16e^{4x} [64x^3 + 240x^2 + 240x + 60]$$

$$\therefore y^5 = [1024e^{4x} x^3 + 3840x^2 e^{4x} + 3840e^{4x} x + 960e^{4x}]$$

If $x^2y'' + xy' + y = 0$ show that

$$x^2y^{n+2} + (2n+1)xy^{(n+1)} + (n^2+1)y^n = 0$$

$$\underset{W_1}{x^2y''} + \underset{W_2}{xy'} + \underset{W_3}{y} = 0$$

$W_1 = x^2y''$

$V = x^2, \quad V^{(1)} = 2x \quad V^2 = 2$

$U = y^{(2)} \quad U' = y^{(3)} \quad U^2 = y^4$

$U^n = y^{(n+2)}$

$$W_1^{(n)} = \sum_{r=0}^{n} {}^nC_r \, U^{(n-r)} V^{(r)}$$

$$= U^n V^{(0)} + nU^{(n-1)}V^{(1)} + \frac{n(n-1)}{2}U^{(n-2)}V^{(2)} \ldots$$

$$W_1^{(n)} = y^{(n+2)}x^2 + n y^{n+1} 2x + \frac{n(n-1)}{2} y^n 2$$

$$= x^2y^{n+2} + 2xny^{(n+1)} + n(n-1)y^n$$

$W_2 = xy'$

$V = x, \quad V^{(1)} = 1$

$U = y^{(1)}, \quad U^{(1)} = y^{(2)}$

$U^{(n)} = y^{(n+1)}$

$$W_2^{(n)} = U^{(n)}V + nU^{(n-1)}V^{(1)}$$

$$= y^{(n+1)}x + n y^{(n)} . 1$$

$$W_2^{(n)} = xy^{(n+1)} + ny^{(n)}$$

$W_3 = y$

$u = y \quad v = 1$

$u^n = y^n$

$W_3 = u^n v = y^n \cdot 1 = y^n$

$W_1^{(n)} + W_2^{(n)} + W_3^{(n)} = 0$

$x^2 y^{(n+2)} + 2n x y^{(n+1)} + (n)(n-1) y^n + x y^{(n+1)} + n y^{(n)}$
$+ y^{(n)} = 0$

$x^2 y^{(n+2)} + x(1+2n)(y)^{n+1} + (n(n-1)+n+1) y^n = 0$

$x^2 y^{(n+2)} + x(1+2n) y^{(n+1)} + (n^2 - n + n + 1) y^n = 0$

$x^2 y^{(n+2)} + x(2n+1) y^{(n+1)} + (n^2+1) y^n = 0$

$x^2 y^{(n+2)} + (2n+1) x y^{(n+1)} + (n^2+1) y^n = 0$