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A

If $y=e^{x^2+x}$ Show that $y''=y'(2x+1)+2y$ and hence prove that $y^{n+2}=(2x+1)y^{(n+1)}+2(n+1)y^n$

Solution

$y=e^{x^2+x}$

$y'=(2x+1)e^{x^2+x}$

$u=2x+1 \quad u'=2$

$v=e^{x^2+x} \quad v'=(2x+1)e^{x^2+x}$

$y''=uv'+v'u$

$y''=(2x+1)[(2x+1)e^{x^2+x}]+e^{x^2+x}2$

but $y'=(2x+1)e^{x^2+x} \quad y=e^{x^2+x}$

$\therefore y''=y'(2x+1)+2y$

$y''-y'(2x+1)-2y=0$

Let $A=y''$

$u=y'' \quad u^n=y^{n+2}$

$v=1 \quad v'=0$

$y^n=u^nv+nv^{n-1}v'$

$v=1 \quad v'=0$

$y^n=u^nv+nu^{n-1}v'$

$y^n=y^{n+2}\cdot1+ny^{n+1}\cdot0$

$y^n=y^{n+2}=A'$

Let $B=y'(2x+1)$

$u=y' \quad u^n=y^{n+1}$

$v=2x+1 \quad v'=2 \quad v''=0$

$y^n=y^{n+1}(2x+1)+ny^n\cdot2$

$y^n=y^{n+1}(2x+1)+2ny^n \Rightarrow B'$

Let $C=2y$

$u=y \quad u^n=y^n$

$v=2 \quad v'=0$

$y^n=2y^n+ny^{n-1}0$

$y^n=2y^n \Rightarrow 0$

$A-B-C\Rightarrow 0$

$A'-B'-C'\Rightarrow 0$

$y^{n+2}-[y^{n+1}(2x+1)+2ny^n]-2y^n=0$

$y^{n+2}=y^{n+1}(2x+1)+2ny^n+2y^n$

$y^{n+2}=y^{n+1}(2x+1)+y^n(2n+2)$

$y^{n+1}=(2x+1)y^{n+1}+(2n+1)y^n$

B

Using the leibnitz theorem, given that

i) $y=x^3e^{4x}$ determine y^5

ii) $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+y=0$, Show that

$x^2yC^{n+2}+(2n+1)xy^{n+1}+(\cancel{2}n^2+1)y^n=0$

Solution

$u=e^{4x} \quad y^n=4^ne^{4x}$

$v=x^3 \quad v'=3x^2 \quad v''=6x \quad v'''=6 \quad v^{iv}=0$

$y^n=4^ne^{4x}x^3+n4^{n-1}e^{4x}(3x^2)+\frac{(n(n-1))4^{n-2}e^{4x}\times6x}{2!}+\frac{n(n-1)(n-2)4^{n-3}e^{4x}6}{3!}+[0]$

$y^n=4^ne^{4x}x^3+3x^2n4^{n-1}e^{4x}+3x(n^2-n)4^{n-2}e^{4x}+(n^3-3n^2+2n)4^{n-3}e^{4x}$

$y^5=1024e^{4x}x^3+3\times256\times5x^2e^{4x}+3(5^2-5)\cdot4^{n-2}\cdot e^{4x}\cdot x+(5^3-3\cdot5^2+2\cdot5)4^{n-3}e^{4x}$

$y^5 = 1024x^3e^{4x} + 3840x^2e^{4x} + 3840$ $\quad y^{n+2}x^2 + y^{n+1}x(2n+1) + y^n(n^2+1) = 0$

$x e^{4x} + 960e^{4x}$

$y^5 = 64e^{4x}x^3\left[16 + \frac{60}{x} + \frac{60}{x^2} + \frac{15}{x^3}\right]$

<u>B</u>

$x^2y'' + xy' + y = 0$

$y^n = u^n v + n u^{n-1} v' + \frac{n(n-1)u^{n-2}v''}{2!}$

$+ \frac{n(n-1)(n-2)}{3!} u^{n-3} v''' + \dots$

$y = uv$

Let $A = x^2y''$

$u = y''$, $u^n = y^{n+2}$

$v = x^2$, $v' = 2x$ $v'' = 2$ $v''' = 0$

$y^n = y^{n+2}x^2 + n y^{n+1} 2x + \frac{n(n-1)y^n \cdot 2}{2!}$

$+ \frac{n(n-1)(n-2)y^{n-1}}{3!} \times 0$

$A' \Rightarrow y^n = y^{n+2}x^2 + n y^{n+1} \cdot 2x + n(n-1)y^n$

Let $B = xy'$

$u = y'$ $u^n = y^{n+1}$

$v = x$ $v' = 1$ $v'' = 0$

$y^n = y^{n+1} \cdot x + n y^{n} + \frac{n(n-1)y^{n-1}}{2} \times 0$

$B' = y^n \Rightarrow y^{n+1}x + n y^n$

Let $C = y$

$C' \Rightarrow y^n$

$A + B + C \Rightarrow 0$

$A' + B' + C' \Rightarrow 0$

$y^{n+2}x^2 + n y^{n+1} \cdot 2x + n(n-1)y^n + y^{n+1}$

$x + n y^n + y^n = 0$