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16/ENG03/027
Civil Engineering
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ENG 381
Engineering Mathematics III

1) If $y = e^{x^2+x}$

show that

$$y'' = y'(2x+1) + 2y$$

and hence prove that

$$y^{(n+2)} = (2x+1)y^{(n+1)} + 2(n+1)y^n$$

Solution

$$\frac{dy}{dx} = (2x+1)e^{x^2+x}$$

$$\frac{d^2y}{dx^2} = u\frac{dv}{dx} + v\frac{du}{dx}$$

$$= 2x+1(2x+1)e^{x^2+x} + 2e^{x^2+x}$$

$$\therefore y'' = y'(2x+1) + 2y$$

using Leibnitz theorem

① $v = 1 \quad u = y''$

② $v = 2x+1 \quad u = y'$

③ $v = 1 \quad u = 2y$

for $y'' = y^{(n+2)} + 0$

for $y'(2x+1) = y^{(n+1)}(2x+1) + ny^n.2 + 0$

for $2y = 2y^n$

$$y^{(n+2)} = y^{(n+2)}(2x+1) + 2ny^n + 2y^n$$

$$y^{(n+2)} = (2x+1)y^{(n+1)} + 2(n+1)y^n$$

2) Using the Leibnitz theorem, given that

(i) $y = x^3 e^{4x}$, determine $y^{(5)}$

Solution

$$y^{(n)} = \sum_{r=0}^{n} {}^nC_r\, u^{(n-r)} v^{(r)}$$

$v' = 3x^2$ $\quad u' = 4e^{4x}$

$v'' = 6x$ $\quad u'' = 16e^{4x}$

$v''' = 6$ $\quad u^{(n)} = 4^n e^{2x}$

$v^{(4)} = 0$

$$y^{(n)} = u^n v + n u^{(n-1)} v^{(1)} + \frac{n(n-1)}{2!} u^{(n-2)} v^{(2)} + \dots$$

$$y^{(n)} = 4^n e^{2x} x^3 + n 4^{n-1} e^{2x}(3x^2) + \frac{n(n-1)}{2!} 4^{n-2} e^{2x}(6x)$$

$$+ \frac{n(n-1)(n-3)}{3!} 4^{n-3} e^{2x}(6) + \frac{n(n-1)(n-2)(n-3)}{4!} 4^{n-4} e^{2x}(0)$$

$$y^{(n)} = e^{2x} 4^{n-3} \left[64x^3 + 16n(3x^2) + n(n-1)12x + n(n-1)(n-2)\right]$$

$$y^{(5)} = 16e^{2x}\left[64x^3 + 240x^2 + 240x + 160\right]$$

$$y^{(5)} = 64e^{2x}\left[16x^3 + 60x^2 + 60x + 15\right]$$

11) $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$, show that $x^2y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2+1)y^{(n)} = 0$

Solution

recall,

$y^{(n)} = u^n v + n u^{n-1} v^{(1)} + \frac{n(n-1)}{2!} u^{n-2} + v^{(2)} + \ldots$

① $v = x^2$, $u = y''$

② $v = x$, $u = y'$

③ $v = 1$, $u = y$

for x^2y'': $y^{(n+2)}x^2 + ny^{(n+1)}.2x + \frac{n(n-1)}{2!} y^n.2$

for xy': $y^{(n+1)}.x + ny^n$

for y: y^n

$= y^{(n+2)}x^2 + 2nxy^{(n+1)} + n(n-1)y^n + y^{(n+1)}x + ny^n + y^n$

$(2n+1)xy^{(n+1)} + (n^2-n+n+1)y^n + x^2y^{(n+2)}$

$= x^2y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2+1)y^{(n)} = 0$

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