$$y^{(n+2)}x^2 + xy^{(n+1)}(2n+1) + (n^2+1)y^n$$

$$= x^2y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2+1)y^n$$

---

1) $y'' = y'(2x+1) + 2y$

$$y'' = y^{(n+2)} + ny^{(n+1)}(0) + \frac{n(n-1)}{2!}y^n(0) + 0$$

$y'(2x+1)$ taking $u = y'$ & $v = 2x+1$

$y'(2x+1) = y^{(n+1)}(2x+1) + ny^n(2) + 0$

$2y = 2y^n + 0$

Adding all terms;

$$y^{(n+2)} = y^{(n+1)}(2x+1) + 2ny^n + 2y^n$$

$$y^{(n+2)} = (2x+1)y^{(n+1)} + 2(n+1)y^n$$

one

Differenting n times using leibnitz theorem

~~$y=y$~~ x^2y''

Taking $u=y''$ & $v=x^2$

$$x^2y'' = {}^nC_0 u^{(n)} v + {}^nC_1 u^{(n-1)} v^{(1)} + {}^nC_2 u^{(n-2)} v^{(2)}$$

$$= (1) y^{(n+2)} x^2 + n y^{(n+1)} \cdot 2x + \frac{n(n-1)}{2!} y^{(n)} 2 + 0$$

$$= y^{(n+2)} \cdot x^2 + n y^{(n+1)} \cdot 2x + \frac{n(n-1)}{2!} y^{(n)} \cdot 2 + 0$$

xy': Taking $u=y'$ & $v=x$

$$xy' = (1) y^{(n+1)} x + n y^{(n)} (1) + 0$$

$$= y^{n+1} x + n y^{(n)} + 0$$

$y. = y^{(n)}$

Adding all the terms

$$y^{(n+2)} x^2 + n y^{(n+1)} 2x + \frac{n(n-1)}{\cancel{2}\times 1} y^{(n)} \cdot \cancel{2} + y^{(n+1)} \cdot x + n y^{(n)} + y^{(n)}$$

$$y^{(n+2)} x^2 + x y^{(n+1)} (2n+1) + y^{(n)} (n(n-1) + n + 1)$$

Enenike M. Winifred
$\cancel{y = x^3 \cdot 4}$ 16/ENG03/025 16/11/18.
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(1) (i) $y = x^3 e^{4x}$
$y = uv$ $u''''' = 1024e^{4x}$ $u'''' = 256e^{4x}$
$u = e^{4x}$ $u' = 4e^{4x}$ $u'' = 16e^{4x}$ $u''' = 64e^{4x}$
$v = x^3$ $v' = 3x^2$ $v'' = 6x$ $v''' = 6$

$y^5 = u^{(5)}v + 5u^{(4)}v^{(1)} + 10u^{(3)}v^{(2)} + 10u^{(2)}v^{(3)} + 5u^{(1)}v^{(4)} + uv^{(5)}$

$\cancel{y^5 = }$

$y^5 = 1024e^{4x} \cdot x^3 + 5 \cdot 256e^{4x} \cdot 3x^2 + 10 \cdot 64e^{4x} \cdot 6x + 10 \cdot 16e^{4x} \cdot 6 + 5 \cdot 4e^{4x} \cdot 0 + e^{4x} \cdot 0$

$y^5 = 1024e^{4x} x^3 + 3840e^{4x} x^2 + 3840e^{4x} x + 960e^{4x}$

$y^5 = e^{4x}(1024x^3 + 3840x^2 + 3840x + 960)$

$y^5 = 64e^{4x}(16x^3 + 60x^2 + 60x + 15)$

(ii) $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

$x^2y'' + xy' + y = 0$