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16/ENG01/005
Chemical Engineering
ENG 381 - Engineering Mathematics III
Assignment 2

Questions

1 If $y = e^{x^2+x}$
show that
$$y'' = y'(2x+1) + 2y$$
and hence prove that; $y^{(n+2)} = (2x+1)y^{(n+1)} + 2(n+1)y^n$

Solution

$y = e^{x^2+x}$ —— ①

using chain rule; let $u = x^2+x \Rightarrow y = e^u$

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

$\frac{dy}{du} = e^u$, $\frac{du}{dx} = 2x+1$

$\Rightarrow \frac{dy}{dx} = e^u(2x+1)$ $\therefore \frac{dy}{dx} = e^{x^2+x}(2x+1)$

$dy/dx = y' \Rightarrow y' = e^{x^2+x}(2x+1)$ —— ②

$\frac{d^2y}{dx^2} = y''$

~~let~~ using product rule, $\frac{dy}{dx} = U\frac{dv}{dx} + V\frac{du}{dx}$

let $u = e^{x^2+x}$, $\frac{du}{dx} \Rightarrow \frac{du}{dy} \times \frac{dy}{dx}$

let $x^2+x = y$ $\therefore \frac{dy}{dx} = 2x+1$

~~let~~ $\Rightarrow u = e^y$ $\therefore \frac{du}{dy} = e^y$

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$\Rightarrow \frac{du}{dx} = \frac{du}{dy} \times \frac{dy}{dx}$

$= e^{y} \times (2x+1)$

$\Rightarrow \frac{du}{dx} = (2x+1)\, e^{x^2+x}$

$\frac{dv}{dx}$ = If $v = 2x+1$

$\frac{dv}{dx} = 2$

$\therefore\ y'' = \frac{d^2y}{dx^2} = (e^{x^2+x})2 + (2x+1)(2x+1)\, e^{x^2+x}$

$y'' = 2(e^{x^2+x}) + (2x+1)(2x+1)\, e^{x^2+x}$

recall from (1) $y = e^{x^2+x}$

$y' = (2x+1)\, e^{x^2+x}$

$\Rightarrow y'' = 2y + (2x+1)y'$

$\Rightarrow \underline{y'' = y'(2x+1) + 2y}$ (Proven) —— (5)

from (3) $y'' = y'(2x+1) + 2y$

$\Rightarrow y^{(2)} = y^{(1)}(2x+1) + 2y$

Let $w_1 = y^{(2)}$

$w_2 = y^{(1)}(2x+1)$

$w_3 = 2y$

a) $w_1 = y^{(2)}$

$\Rightarrow u = y^{(2)}$, $v = 1$

$\therefore u^n = y^{n+2}$, $v' = 0$

hence, $w_1{}^n = u^n v^0 + {}^nC_1 \cdot u^{n-1} v' + {}^nC_2 \cdot u^{n-2} v^2 + \ldots$

$\Rightarrow w_1{}^n = u^n v^0 + nu^{n-1} v' + \frac{n(n-1)}{2} u^{n-2} v^2$

$w_1{}^n = y^{n+2} \cdot 1 + ny^{n-1+2} \cdot 0$

$\Rightarrow w_1{}^n = y^{n+2}$

b) $w_2 = y^{(1)}(2x+1)$

$u = y'$, $v = 2x+1$

$u' = y''$, $v' = 2$, $v'' = 0$

$\Rightarrow u^n = y^{n+1}$

$w_2^n = u^n v^0 + n u^{n-1} v' + \frac{n(n-1)}{2} u^{n-2} v^2$

$w_2^n = y^{n+1}(2x+1) + n(y^{n+1-1})2 + \frac{n(n-1)}{2} y^{n-2+1} \cdot 0$

$w_2^n = (2x+1) y^{n+1} + 2n y^n$

c) $w_3 = 2y$

$u = y$, $v = 2$

$u' = y'$, $v' = 0$

$\Rightarrow u^n = y^n$

hence, $w_3^n = u^n v^0 + n u^{n-1} v' + \frac{n(n-1)}{2} y^{n-2} v^2$

$w_3^n = y^n . 2 + n y^{n-1} . 0$

$\Rightarrow w_3^n = 2y^n$

hence, $w_1^n = w_2^n + w_3^n$

$y^{(n+2)} = (2x+1) y^{(n+1)} + 2ny^n + 2y^n$

$y^{(n+2)} = (2x+1) y^{(n+1)} + 2(n+1) y^n$ (Proven)

Question 2

$y = x^3 e^{4x}$

$v = x^3$, $u = e^{4x}$

$v' = 3x^2$, $u' = 4e^{4x}$ $\therefore u^n = 4^n e^{4x}$

$v'' = 6x$, $u'' = 16e^{4x}$

$v''' = 6$, $u''' = 64e^{4x}$

$y^n = {}^nC_0 u^n v^0 + {}^nC_1 u^{n-1} v^1 + {}^nC_2 u^{n-2} v^2 + {}^nC_3 u^{n-3} v^3$

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$y^n = u^n v^0 + n u^{n-1} v^1 + \frac{n(n-1)}{2!} v^2 u^{n-2} + \frac{n(n-1)(n-2)}{3!} u^{n-3} v^3$

$y^n = u^n v^0 + n u^{n-1} v^1 + \frac{n(n-1) u^{n-2} v^2}{2!} + \frac{n(n-1)(n-2) u^{n-3} v^3}{3!}$

where $n = 5$

$y^{(5)} = (4^5 e^{4x} \cdot x^3) + (5 \cdot 4^{5-1} e^{4x} \cdot 3x^2) + \frac{5(5-1) 4^{5-2} e^{4x} \cdot 6x}{2 \times 1}$

$+ \frac{5(5-1)(5-2) 4^{5-3} e^{4x} \cdot 6}{3 \times 2 \times 1}$

$y^{(5)} = 1024 e^{4x} x^3 + 3840 e^{4x} x^2 + 3840 e^{4x} x + 960 e^{4x}$

$\Rightarrow y^{(5)} = 64 e^{4x} (16x^3 + 60x^2 + 60x + 15)$

ii) $x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + y = 0$

show that $x^2 y^{(n+2)} + (2n+1) x y^{(n+1)} + (n^2+1) y^{(n)} = 0$

$\Rightarrow x^2 y'' + xy' + y = 0$

a) let $w_1 = x^2 y''$ so that, $w_1^n + w_2^n + w_3^n = 0$ —— *

$v = x^2$, $u = y''$

$v' = 2x$, $u' = y'''$

$v'' = 2$, $u'' = y^{iv}$

$\Rightarrow u^n = y^{n+2}$

$w_1^n = u^n v^0 + n u^{n-1} v' + \frac{n(n-1)}{2} u^{n-2} v^2$

$w_1^n = y^{n+2} \cdot x^2 + n(y^{n+2-1}) \cdot 2x + \frac{n(n-1)}{\cancel{2}} y^{n+2-2} \cdot \cancel{2}$

$w_1^n = x^2 (y^{n+2}) + 2nx(y^{n+1}) + n(n-1) y^n$

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b) Let $w_2 = xy'$

$v = x$, $u = y'$

$v' = 1$, $u' = y''$

$\Rightarrow u^n = y^{n+1}$

$w_2^n = u^n v^0 + n u^{n-1} v' + \frac{n(n-1)}{2} u^{n-2} v^2$

$w_2^n = (y^{n+1}) x + n(y^{n+1-1}) . 1$

$w_2^n = xy^{n+1} + ny^n$

c) Let $w_3 = y$

$u = y$, $v = 1$

$u' = y'$, $v' = 0$

$\Rightarrow u^n = y^n$

$w_3^n = u^n v^0 + n u^{n-1} v'$

$w_3^n = y^n . 1 + n y^{n-1} . 0$

$\therefore w_3^n = y^n$

recall from (*) $w_1^n + w_2^n + w_3^n = 0$

$\therefore x^2 y^{n+2} + 2nx(y^{n+1}) + n(n-1)y^n + xy^{n+1} + ny^n + y^n = 0$

factorising: $x^2 y^{n+2} + 2nx(y^{n+1}) + xy^{n+1} + n(n-1)y^n + ny^n + y^n = 0$

$x^2 y^{(n+2)} + xy^{n+1}(2n+1) + y^n(n^2 - n + n + 1) = 0$

$\therefore$

$\therefore x^2 y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2+1)y^{(n)} = 0$ (Proven)

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