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16/ENG01/002
CHEMICAL ENGINEERING

a) $y = e^{x+x^2}$

Using the chain rule

Let $u = x + x^2 \therefore y = e^u$

$du/dx = 1 + 2x \qquad dy/du = e^u$

$\therefore \frac{dy}{du} \cdot \frac{du}{dx} = (1+2x) e^u = y'$

$y' = (2x+1) e^{(x+x^2)}$

$y'' = \frac{d^2y}{dx^2} = (2x+1)\left((2x+1) e^{x^2+x}\right) + 2\left(e^{x^2+x}\right)$

$y'' = (2x+1)\left((2x+1)e^{x^2+x}\right) + 2\left(e^{x^2+x}\right)$

where $e^{x^2+x} = y$

also $y' = (2x+1) e^{(x+x^2)}$

$\therefore y'' = y'(2x+1) + 2y$ //

b) $y'' = y'(2x+1) + 2y$

$y'' - y'(2x+1) - 2y = 0$

Using Leibnitz Theorem

$W_1^n = y''$ $\qquad V_1 = 1 \qquad U^0 = y''$

$V_1' = 0 \qquad U^n = y^{n+2}$

$W_1^n = U^n V + nU^{n-1}V' + \frac{n(n-1)}{2!} U^{n-2} V'' + \ldots$

$W_1^n = y^{n+2}(1) + ny^{n+1}(0) + \ldots$

$W_1^n = y^{n+2}$

Expanding $-y'(2x+1)$ $\quad W_2^n = -2xy'$

$W_2^n = -2xy'$

$W_3 = -y'$

$W_2^n = -2xy'$

$y^{(0)} = -2x \qquad u = y'$

$y' = -2 \qquad u' = y''$

$y'' = 0 \qquad u^n = y^{n+1}$

$W_2^n = y^n u + n y^{n-1} u' + \frac{n(n-1)}{2!} y^{n-2} u'' + \dots$

$W_2^n = y^{n+1}(-2x) + n y^n (-2) + \frac{n(n-1)}{2!} \times y^{n-1}(0)$

$W_2^n = -2xy^{n+1} - 2ny^n$

$W_3^n = -y'$

$y^0 = y' \qquad v = -1$

$y' = y'' \qquad v' = 0$

$y^n = y^{n+1}$

$W_3^n = y^{n+1}(-1) + 0 = -y^{n+1}$

$W_4^n = -2y \qquad u = y \qquad v = -2$

$u' = y' \qquad v' = 0$

$u^n = y^n$

$W_4^n = -2y^n$

Adding all results

$W_1^n + W_2^n + W_3^n + W_4^n = y^{n+2} - 2xy^{n+1} - 2ny^n - (y^{n+1}) - 2y^n = 0$

$y^{n+2} - 2xy^{n+1} - 2ny^n - y^{n+1} - 2y^n = 0$

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$$y^{n+2} = (2x+1)y^{n+1} + 2(n+1)y^n$$

2) $y = x^3 e^{4x}$ determine y^5

Let $u = e^{4x}$, and $v = x^3$

$u' = 4e^{4x}$ $\quad v' = 3x^2$

$u'' = 16e^{4x}$ $\quad v'' = 6x$

$u^n = 4^n e^{4x}$ $\quad v''' = 6$

$v^{iv} = 0$

$$y^n = u^n v + n u^{n-1} v' + \frac{n(n-1)}{2!} u^{n-2} v^2 + \frac{n(n-1)(n-2)}{3!} \cdot u^{n-3} v^3 + \dots$$

$$y^n = 4^n e^{4x} + n4^{n-1} e^{4x} 3x^2 + \frac{n(n-1)}{2!} 4^{n-2} e^{4x} 6x + \frac{n(n-1)(n-2)}{6} \times 4^{n-3} e^{4x} 6.$$

$$\therefore y^n = 4^n e^{4x} x^3 + n4^{n-1} e^{4x} 3x^2 + n(n-1)4^{n-2} 3x + n(n-1)(n-2)4^{n-3} e^{4x} \dots$$

$$y^n = 4^{n-3} e^{4x} (4^3 x^3 + n4^2 \times 3x^2 + n(n-1) 4 \times 3x + n(n-1)(n-2))$$

$$y^n = 4^{n-3} e^{4x} (64x^3 + n48x^2 + n(n-1)12x + n(n-1)(n-2))$$

Therefore

$$\therefore y^5 = 4^{5-3} e^{4x} (64x^3 + 5(48)x^2 + 5(4)12x + 5(4)(3))$$

$$y^5 = 16e^{4x}(64x^3 + 240x^2 + 240x + 60)$$

$$\therefore y^5 = (1024e^{4x}x^3 + 3840x^2e^{4x} + 3840e^{4x}x + 960e^{4x})$$

If $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$, show that

$$x^2 y^{n+2} + (2n+1) x y^{(n+1)} + (n^2+1) y^n = 0$$

$$x^2 y'' + x y' + y = 0$$

$$\downarrow \qquad \downarrow \qquad \downarrow$$

$$W_1 \qquad W_2 \qquad W_3$$

$W_1 = x^2 y''$

$V = x^2, \quad V' = 2x, \quad V'' = 2$

$u = y^2, \quad u' = y^3, \quad u^2 = y^4$

$$u^n = y^{n+2}$$

$$W_1^{(n)} = \sum_{r=0}^{n} {}^nC_r \, u^{(n-r)} V^r$$

$$= u^n V^0 + n u^{n+1} V' + \frac{n(n-1)}{2} u^{(n-2)} V^2 \ldots$$

$$W_1^{(n)} = y^{n+2} x^2 + n y^{n+1} 2x + \frac{n(n-1)}{\cancel{2}} y^n \cancel{2}$$

$$= x^2 y^{n+2} + 2xn y^{(n+1)} + n(n-1) y^n$$

$W_2 = x y', \quad V = x, \quad V' = 1$

$u = y', \quad u' = y^2$

$u^n = y^{(n+1)}$

$$W_2^{(n)} = u^{(n)} V + n u^{(n-1)} V'$$

$$= y^{(n+1)} x + n y^n \cdot 1$$

$$W_2^n = x y^{n+1} + n y^n$$

$W_3 = y$

$u = y, \quad y^n = y^n$

$$W_3 = u^n V = y^n \cdot 1 = y^n$$

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$$W_1^{(n)} + W_2^{(n)} + W_3^{(n)} = 0$$

$$x^3 y^{(n+2)} + 2n\, xy^{(n+1)} + (n)(n-1)\, y^n + xy^{(n+1)} + ny^{(n)} + y^{(n)} = 0$$

$$x^2 y^{(n+2)} + x(1+2n)(y)^{n+1} + (n(n-1) + n + 1)\, y^n = 0$$

$$x^2 y^{(n+2)} + x(1+2n)\, y^{(n+1)} + (n^2 - n + n + 1)\, y^n = 0$$

$$x^2 y^{(n+2)} + x(2n+1)\, y^{(n+1)} + (n^2+1)\, y^n = 0$$

$$x^2 y^{(n+2)} + (2n+1)\, xy^{(n+1)} + (n^2+1)\, y^n = 0$$