IROEGBU LAWRENCE PRINCE
16/ENG02/024
COMPUTER ENGINEERING
ENG 381 Assignment (2)

Answers.

1) $y = e^{x^2} + x$

$\ln y = x^2 + x$

Differentiating both sides.

$\frac{1}{y}\frac{dy}{dx} = 2x + 1$

multiply both sides by y.

$\frac{dy}{dx} = [2x+1]y$

$\frac{d^2y}{dx^2} = U\frac{dv}{dx} + V\frac{du}{dx}$

$V = 2x+1$ $\qquad dv/dx = 2$

$U = y$ $\qquad du/dx = \frac{du}{dy} \cdot \frac{dy}{dx} = 1 \cdot \frac{dy}{dx}$

So that

$\frac{d^2y}{dx^2} = (2x+1) \cdot \frac{1dy}{dx} + 2y$

$\frac{d^2y}{dx^2} = (2x+1)\frac{dy}{dx} + 2y$

$y'' = y'(2x+1) + 2y$

$\Rightarrow y'' = y'(2x+1) + 2y$

$y^{(2)} = y^{(1)}(2x+1) + 2y$

2) Differentiating $y^{(1)}(2x+1)$

Let $V = 2x+1$ $\qquad U = y'$

$V' = 2$ $\qquad U^n = y^{(n+1)}$

$V^2 = 0$ $\qquad U^{(n-1)} = y^n$

Recall from Leibnitz theorem

$U^nV + nU^{(n-1)}V' + \cancel{\frac{n(n-1)U^{(n-2)}V^2}{2!}}$ $\rightarrow 0$ $\quad$ But $V^2 = 0$

$= y^{(n+1)} \cdot (2x+1) + n(y^{n}) \cdot 2$

Differentiating $y^{(2)}$ we have; $y^{(n+2)}$

$(2y = 2y^{n})$

$y^{(n+2)} = y^{(n+1)} \cdot (2x+1) + 2ny^{n} + 2y^{n}$

$y^{(n+2)} = (2x+1)y^{(n+1)} + 2y^{n}(n+1)$

Proved.

2) Using Leibnitz theorem, given that

(i) $y = x^3 e^{4x}$, determine $y^{(5)}$

$u = e^{4x}$ $\quad v = x^3$

$u^{n} = 4^{n} e^{4x}$ $\quad v' = 3x^2$

$u^{(n-1)} = 4^{(n-1)} e^{4x}$ $\quad v^2 = 6x$

$u^{(n-2)} = 4^{(n-2)} e^{4x}$ $\quad v^3 = 6$

$u^{(n-3)} = 4^{(n-3)} e^{4x}$ $\quad v^4 = 0$

where $n = 5$

$4^5 e^{4x} = 1024 e^{4x}$

$u^{(n-1)} = 4^{(5-1)} e^{4x} = 256 e^{4x}$

$u^{(n-2)} = 4^{(5-2)} e^{4x} = 64 e^{4x}$

$u^{(n-3)} = 4^{(5-3)} e^{4x} = 16 e^{4x}$

$y^{(5)} = (1024 e^{4x} \cdot x^3) + n(256 e^{4x} \cdot 3x^2) \cancel{+ n(\ldots) \; 64 e^{4x} \cdot 6x}$

$+ \frac{n(n-1) 64 e^{4x} \cdot 6x}{2!} + \frac{n(n-1)(n-2)(16 e^{4x}) \cdot 6}{3!}$

We have

$= (1024x^3 + 3840x^2 + 3840x + 960) e^{4x}$

$= y^{(5)} = e^{4x}(1024x^3 + 3840x^2 + 3840x + 960)$

2(ii) $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

Show that $x^2 y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2+1)y^{n} = 0$

$x^2y'' + xy' + y = 0$

$u = y^{(2)}$ $\qquad v = x^2$

$u^n = y^{(n+2)}$ $\qquad v' = 2x$ $\Rightarrow$ $\boxed{y^{(n+2)}(x^2) + n(y^{n+1})2x + \frac{n(n-1)y^n(2)}{2!}}$ Since $v^3 = 0$

$u^{(n-1)} = y^{(n+1)}$ $\qquad v^2 = 2$

$u^{(n-2)} = y^n$ $\qquad v^3 = 0$

For (xy')

$v = x \qquad u = y'$

$v' = 1 \qquad u^n = y^{(n+1)}$

$v^2 = 0 \qquad u^{(n-1)} = y^n$

Applying

$y^n = u^n v + n u^{(n-1)} v^{(1)} + \cancel{\frac{n(n-1) u^{(n-2)}}{2!} v^2}^{\,0} + \ldots$

Since $v^2 = 0$

For (xy')

$y^n = y^{(n+1)} \cdot x + n y^{(n)}$

$\Rightarrow y^n = x^2 y^{(n+2)} + 2xn y^{(n+1)} + n(n-1) y^n + x y^{(n+1)} + n y^n + y^n.$

then

$y^n = x^2 y^{(n+2)} + x y^{(n+1)} \cdot (2n+1) + y^n(n^2 - n + n + 1) = 0$

$= x^2 y^{(n+2)} + x y^{(n+1)}(2n+1) + y^n(n^2+1) = 0$

$\Rightarrow y^n = x^2 y^{(n+2)} + x y^{(n+1)}(2n+1) + y^n(n^2+1) = 0$