NDUBUISI DIVINE EBUBE
17/ENG02/089
COMPUTER ENGINEERING

1.) $y = e^{x} + x^2$

Using theorem Rule / function of a function

$y = e^{u}$ $\quad$ $y = eu$

$u = x + x^2$ $\quad$ $dy/du = e^{u}$

$\frac{dy}{dx} = 1 + 2x$

$dy/du \times du/dx = (2x+1)e^{u}$

$dy/dx = (2x+1)e^{(x+x^2)}$

$\therefore y' = (2x+1)(e^{x}+x^2)$

$y'' = \left(\frac{d^2y}{dx^2}\right) = [2x+1]\left[(2x+1)e^{x^2}+x\right] + 2\left[e^{x^2+x}\right]$

$y'' = (2x+1)\left[(2x+1)e^{x^2+x}\right] + 2\left[e^{x^2+x}\right]$

where $e^{x^2+x} = y$

and $(2x+1)(e^{x^2+x}) = y'$

$\therefore y'' = y'(2x+1) + 2(y)$

(b.) $y'' = y'(2x+1) + 2y$

$y'' - y'(2x+1) - 2y = 0$

Using Leibnitz theorem.

$W_1^{n} = y''$ $\quad$ $V_1 = 1$ $\quad$ $U^{0} = y''$

$V_1 = 0$ $\quad$ $U^{n} = y^{n+2}$

$W_1^{n} = U^{n}V + nU^{n-1}V' + \frac{n(n-1)}{2!}U^{n-2}V'' + \ldots$

$W_1^{n} = y^{n+2}(1) + ny^{n-1}(0) + \ldots$

$\therefore W_1^{n} = y^{n+2}$

Expanding $y'(2x+1)$

$W_2^n = 2xy'$

$v^{(0)} = -2x$ $\quad$ $u = y'$

$v' = -2$ $\quad$ $u' = y''$

$v'' = 0$ $\quad$ $u^n = y^{n+1}$

$W_2^n = u^n v + n u^{n-1} v' + \frac{n(n-1)}{2!} u^{n-2} v'' + \dots$

$W_2^n = y^{n+1}(-2x) + n y^n (-2) + \frac{n(n-1)}{2!} \times y^{n-1}(0)$

$W_2^n = -2xy^{n+1} - 2ny^n$

$W_3^n = -y'$

$u^0 = y'$ $\quad$ $v = -1$

$u' = y''$ $\quad$ $v' = 0$

$u^n = y^{n+1}$

$W_3^n = y^{n+1}(-1) + 0 = -y^{n+1}$

$W_4^n = -2y$

$u = y$ $\quad$ $v = 2$

$u' = y'$ $\quad$ $v' = 0$

$u^n = y^n$

$W_4^n = -2y^n$

Adding all results

$W_1^n + W_2^n + W_3^n + W_4^n = y^{n+2} - 2xy^{n+1} - 2ny^n - (y^{n+1}) - 2y^n = 0$

$y^{n+2} - 2xy^{n+1} - 2ny^n - y^{n+1} - 2y^n = 0$

$y^{n+2} = 2xy^{n+1} + 2ny^n + y^{n+1} + 2y^n$

$y^{n+2} = (2x+1)y^{n+1} + 2(n+1)y^n$

$x^3 e^{4x}$

$u = e^{4x}$ $\quad$ $v = x^3$

$u' = 4e^{4x}$ $\quad$ $v' = 3x^2$

$u'' = 16e^{4x}$ $\quad$ $v'' = 6x$

$u^n = 4^n e^{4x}$ $\quad$ $v''' = 6$

$v^{iv} = 0$

$= W^n \; u^n v + n u^{n-1} v' + \frac{n(n-1)}{2!} u^{n-2} v^2 + \frac{n(n-1)(n-2)}{3!}$

$4^n e^{4x} x^3 + n 4^{n-1} e^{4x} 3x^2 + \frac{n(n-1)}{2!} 4^{n-2} e^{4x} 6x + \frac{n(n-1)(n-2)}{6} \times 4^{n-3} e^4$

$= 4^n e^{4x} x^3 + n^{4^{n-1}} e^{4x} 3x^2 + n(n-1) 4^{n-2} 3x + n(n-1)(n-2) 4^{n-3} e^{4x}$

$= 4^{n-3} e^{4x} [4^3 x^3 + n 4^2 + 3x^2 + n(n-1) 4 \times 3x + n(n-1)(n-2)]$

$= 4^{n-3} e^{4x} [64x^3 + n 48x^2 + n(n-1) 12x + n(n-1)(n-2)]$

$= 4^{5-3} e^{4x} [64x^3 + 5(48)x^2 + 5(4)12x + 5(4)(3)]$

$16 e^{4x} [64x^3 + 240x^2 + 240x + 60]$

$y^5 = [1024 e^{4x} x^3 + 3840 x^2 e^{4x} + 3840 e^4 + 960 e^{4x}]$

11) $x^2 y'' + xy' - y - 0$

$x^2 y^{n+2} + (2n+1) xy^{(n+1)} + (n+1) y^n = 0$

$x^2 y'' + xy' + y = 0$

$W_1 = x^2 y''$

$v = x^2 \qquad v' = 2x \qquad v^2 = 2$

$u = y^{(2)} \qquad u' = y^{(3)} \qquad u^2 = y^4$

$u^n = y^{(n+3)}$

$W_1^{(n} = \sum_{r=0}^{n} {}^nC_r \, u^{(n-r)} v$

$= u^n v^{(0)} + n u^{(n+1)} v' + \frac{n(n-1)}{2} u^{(n-2)} v^2$

$W^{(n)} = y^{(n+2)} x^2 + n y^{n+1} 2x + \frac{n(n-1)}{2} y^n 2$

$= x^2 y^{n+2} + 2xn y^{(n+1)} + n(n-1) y^n$

$W_2 = xy'$

$v = x_1 \qquad v' = 1$

$u = y' \qquad u' = y^{(2)}$

$$u^{(n)} = y^{(n+1)}$$

$$W_2 = u^n v + n u^{(n-1)} v'$$

$$= y^{(n+1)} x + n y^{(n)}$$

$$W_2^{(n)} = x y^{(n+1)} + n y^{(n)}$$

$$W_3 = y$$

$$u = y \quad v = 1$$

$$u^n = y^n$$

$$W_3 = u^n v = y^n \quad 1 = y^n$$

$$W_1^{(n)} + W_2^{(n)} + W_3^{(n)} = 0$$

$$x^2 y^{(n+2)} + 2n x y^{(n+1)} + (n)(n-1) y^n + x y^{(n-1)} + y^{(n)} = 0$$

$$x^2 y^{(n+2)} + x(1+2n) y^{(n+1)} + (n^2 - n + n + 1) y^n = 0$$

$$x^2 y^{(n+2)} + x(2n+1) y^{(n+1)} + (n^2+1) y^n = 0$$

$$x^2 y^{(n+2)} + (2n+1) x y^{(n+1)} + (n^2+1) y^n = 0$$

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