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16/ENG06/006

Mechanical

ENG 381 Assignment

1 If $y = e^{x^2+x}$ show that $y'' = y'(2x+1) + 2y$ and prove that

$y^{(n+2)} = (2x+1)\, y^{n+1} + 2(n+1) y^n$.

solution

$Y = e^{x^2+x}$

$Y' = (2x+1)\, e^{x^2+x}$

$u = 2x+1 \qquad v = e^{x^2+x}$

$\frac{du}{dx} = 2 \qquad \frac{dv}{dx} = (2x+1) e^{x^2+x}$

$Y'' = u\frac{dv}{dx} + v\frac{du}{dx}$

$Y'' = (2x+1)(2x+1)\, e^{x^2+x} + e^{x^2+x}(2)$

$Y'' = (2x+1) Y' + 2Y$

$\therefore Y'' = Y'(2x+1) + 2Y$

Using Leibnitz theorem

$Y^n = u^n v + n u^{n-1} v' + \frac{n(n-1)}{2!} u^{n-2} v^2 + \frac{n(n-1)(n-2)}{3!} u^{n-3} v^3 + \ldots$

From $Y'' = Y'(2x+1) + 2Y$

$Y'' - Y'(2x+1) - 2Y = 0$

Let $A = Y''$

$u = Y''$

$u^n = Y^{n+2}$

Let $B = Y'(2x+1)$

$u = Y' \qquad v = 2x+1$

$u^n = Y^{n+1} \qquad v' = 2$

$v'' = 0$

$Y^n = Y^{n+1}(2x+1) + n Y^{n+1-1}(2) + \frac{n(n-1)}{2!} Y^{n+1-2}(0)$

$Y^n = Y^{n+1}(2x+1) + 2n Y^n$

Let $c = 2y$

$u = y$

$u^n = y^n$

$\therefore y^n = y^n(2) + 0$

$y^n = 2y^n$

$y^n = A - B - C$

$y^{n+2} - \left(y^{n+1}(2x+1) + 2ny^n\right) - 2y^n = 0$

$y^{n+2} - y^{n+1}(2x+1) - 2ny^n - 2y^n = 0$

$y^{n+2} = y^{n+1}(2x+1) + 2ny^n + 2y^n$ -

$y^{n+2} = y^{n+1}(2x+1) + 2(n+1)y^n$ //

2 Using the Leibnitz theorem given that

i $y = x^3 e^{4x}$, determine $y^{(5)}$

ii $x^2 \dfrac{d^2y}{dx^2} + x\dfrac{dy}{dx} + y = 0$

solution

i $y = x^3 e^{4x}$

$$y^n = u^n v + nu^{n-1}v' + \frac{n(n-1)}{2!} u^{n-2} v^2 + \frac{n(n-1)(n-2)}{3!} u^{n-3} v^3 + \dots$$

Let $v = x^3$ $\quad u = e^{4x}$

$v' = 3x^2$ $\quad u' = 4e^{4x}$

$v'' = 6x$ $\quad u'' = 16e^{4x}$

$v''' = 6$ $\quad u''' = 64e^{4x}$

$v^{iv} = 0$ $\quad u^{iv} = 256e^{4x}$

$v^v = 0$ $\quad u^v = 1024e^{4x}$

$$y^5 = u^5 v + 5u^{5-1}v' + \frac{5(5-1)}{2!} u^{5-2} v^2 + \frac{5(5-1)(5-2)}{3!} u^{5-3} v^3 +$$

$$\frac{5(5-1)(5-2)(5-3)}{4!} u^{5-4} v^4 + \frac{5(5-1)(5-2)(5-3)(5-4)}{5!} u^{5-5} v^5$$

$$y^5 = u^5 v + 5u^4 v' + 10u^3 v^2 + 10u^2 v^3 + 5u' v^4 + uv^5$$

$$y^5 = 1024e^{4x}x^3 + 5(256e^{4x})3x^2 + 10(64e^{4x})6x + 10(16e^{4x})6 + 5(4e^{4x})(0) + e^{4x}(0)$$

$$y^5 = 1024x^3e^{4x} + 3840x^2e^{4x} + 3840xe^{4x} + 960e^{4x}$$

iii $$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$$

solution

$$x^2y'' + xy' + y = 0$$

using Leibnitz theorem;

$$y^n = u^nv + nu^{n-1}v' + \frac{n(n-1)}{2!}u^{n-2}v^2 + \frac{n(n-1)(n-2)}{3!}u^{n-3}v^3 + \dots$$

Let $A = x^2y''$

$u = y''$ $\quad v = x^2$ $\quad v''' = 0$

$u^n = y^{n+2}$ $\quad v' = 2x$

$v'' = 2$

$$y^n = y^{n+2}x^2 + ny^{n+2-1}2x + \frac{n(n-1)}{2!}y^{n+2-2}(2) + \frac{n(n-1)(n-2)}{3!}y^{n+2-3}(0)$$

$$y^n = y^{n+2}x^2 + ny^{n+1}2x + n(n-1)y^n$$

Let $B = xy'$

$u = y'$ $\quad v = x$ $\quad v'' = 0$

$u^n = y^{n+1}$ $\quad v' = 1$

$$y^n = y^{n+1}x + ny^{n+1-1}(1) + \frac{n(n-1)}{2!}y^{n+1-2}(0)$$

$$y^n = y^{n+1}x + ny^n$$

Let $C = y$

$u^n = y^n$

$y^n = A + B + C$

$$y^n = y^{n+2}x^2 + ny^{n+1}2x + n(n-1)y^n + y^{n+1}x + ny^n + y^n$$

$$y^n = y^{n+2}x^2 + y^{n+1}(2nx + x) + y^n(n(n-1) + n + 1)$$

$$y^n = y^{n+2}x^2 + y^{n+1}x(2n+1) + y^n(n^2 - n + n + 1)$$

$$\therefore y^n = y^{n+2}x^2 + y^{n+1}x(2n+1) + y^n(n^2+1) = 0$$