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161ETG06/006

Mechanical

ETG 381 Assignment

1 If $y = e^{2x+x}$ show that $y'' = y'(2x+1) + 2y$ and prove that $y^{(n+2)} = (2x+1)y^{(n+1)} + 2(n+1)y^n$

Solution

$$y = e^{2x+x}$$

$$y' = (2x+1)e^{2x+x}$$

$$u = 2x+1 \quad v = e^{2x+x}$$

$$\frac{du}{dx} = 2 \quad \frac{dv}{dx} = (2x+1)e^{2x+x}$$

$$y'' = u \frac{dv}{dx} + v \frac{du}{dx}$$

$$y'' = (2x+1)(2x+1)e^{2x+x} + e^{2x+x}(2)$$

$$y'' = (2x+1)y' + 2y$$

$$\therefore y'' = y'(2x+1) + 2y$$

Using Leibnitz theorem

$$y^n = u^n v + n u^{n-1} v' + n(n-1)u^{n-2} v'' + n(n-1)(n-2)u^{n-3} v''' + \dots$$

$$\text{From } y'' = y'(2x+1) + 2y$$

$$y'' - y'(2x+1) - 2y = 0$$

$$\text{Let } A = y''$$

$$u^n = y''$$

$$u^n = y^{n+2}$$

$$\text{Let } B = y'(2x+1)$$

$$u = y' \quad v = 2x+1$$

$$u^n = y^{(n+1)} \quad (1) \quad v' = 2 \quad v(1-2) = 2x+1 \quad v^2 + v^3 + \dots$$

$$v'' = 0$$

$$y^n = y^{(n+1)}(2x+1) + n y^{(n+1-1)}(2) + n(n-1) y^{(n+1-2)}(0)$$

$$y^n = y^{(n+1)}(2x+1) + 2n y^n$$

$$\text{Let } c = 24$$

$$u = y$$

$$u^n = y^n$$

$$\therefore y^n = y^n(a) + 0$$

$$y^n = 24^n$$

$$y^n = A - B - C$$

$$y^{n+2} - (y^{n+1}(2x+1) + 2xy^n) - 24^n = 0$$

$$y^{n+2} - y^{n+1}(2x+1) - 2xy^n - 24^n = 0$$

$$y^{n+2} = y^{n+1}(2x+1) + 2xy^n + 24^n$$

$$y^{n+2} = y^{n+1}(2x+1) + 2(n+1)y^n$$

2 Using the Leibnitz theorem given that

i $y = x^3 e^{4x}$, determine $y^{(5)}$

ii $x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + y = 0$

solution

i $y = x^3 e^{4x}$

$$y^n = uv + nu^{n-1}v' + n(n-1)u^{n-2}v'^2 + n(n-1)(n-2)u^{n-3}v'^3 + \dots$$

Let $v = x^3$

$$u = e^{4x}$$

$$v' = 3x^2$$

$$u' = 4e^{4x}$$

$$v'' = 6x$$

$$u'' = 16e^{4x}$$

$$v''' = 6$$

$$u''' = 64e^{4x}$$

$$v^{(4)} = 0$$

$$u^{(4)} = 256e^{4x}$$

$$v^{(5)} = 0$$

$$u^{(5)} = 1024e^{4x}$$

$$y^5 = u^5 v + 5u^4 v' + 5(5-1)u^3 v'^2 + 5(5-1)(5-2)u^2 v'^3 + \dots$$

$$5(5-1)(5-2)(5-3)u^4 v'^4 + 5(5-1)(5-2)(5-3)(5-4)u^3 v'^5 + \dots$$

$$y^5 = u^5 v + 5u^4 v' + 10u^3 v'^2 + 10u^2 v'^3 + 5u v'^4 + u v'^5$$

$$y^5 = 1024e^{4x}x^3 + 5(256e^{4x})3x^2 + 10(64e^{4x})6x + 10(16e^{4x})6 + 5(4e^{4x})(0) + e^{4x}(0)$$

$$y^5 = 1024x^3e^{4x} + 3840x^2e^{4x} + 3840xe^{4x} + 960e^{4x}$$

ii) $x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + y = 0$

solution

$$x^2 y'' + x y' + y = 0$$

using Leibnitz theorem,

$$y^n = u^n v + n u^{n-1} v' + n(n-1) u^{n-2} v'' + \dots + n(n-1)(n-2) u^{n-3} v''' + \dots$$

Let $A = x^2 y''$

$$v = x^2 \quad v''' = 0$$

$$u = y'$$

$$v' = 2x$$

$$v'' = 2$$

$$y^n = y^{n+2} x^2 + n y^{n+2-1} 2x + n(n-1) y^{n+2-2} (2) + n(n-1)(n-2) \dots$$

$$y^{n+2-3} (0)$$

$$y^n = y^{n+2} x^2 + n y^{n+1} 2x + n(n-1) y^n$$

Let $B = x y'$

$$v = x \quad v'' = 0$$

$$u = y^{n+1} \quad v' = 1$$

$$y^n = y^{n+1} x + n y^{n+1-1} (1) + n(n-1) y^{n+1-2} (0) \dots$$

$$y^n = y^{n+1} x + n y^n$$

Let $C = y$

$$u^n = y^n$$

$$y^n = A + B + C$$

$$y^n = y^{n+2} x^2 + n y^{n+1} 2x + n(n-1) y^n + y^{n+1} x + n y^n + y^n$$

$$A_0 = A_{0,0}x^0 + A_{0,1}x^1 + \dots$$

5)

$$A_1 = A_{1,0}x^0 + A_{1,1}x^1 + A_{1,2}x^2 + \dots + A_{1,n-1}x^{n-1} + A_{1,n}x^n + \dots$$

$$A_2 = A_{2,0} + A_{2,1}x + A_{2,2}x^2 + \dots$$

$$A_3 = A_{3,0} + A_{3,1}x + A_{3,2}x^2 + A_{3,3}x^3 + \dots$$

$$\text{prop } B = xA_1$$

$$A_0 = A_{0,0}x^0 + A_{0,1}x^1 + A_{0,2}x^2 + \dots + A_{0,n-1}x^{n-1} + A_{0,n}x^n + \dots$$

$$A_{0+n-1} = 0$$

5)

$$A_1 = A_{1,0}x^0 + A_{1,1}x^1 + A_{1,2}x^2 + \dots + A_{1,n-1}x^{n-1} + A_{1,n}x^n + \dots + A_{1,2n-1}x^{2n-1} + \dots$$

3)

$$A_{1+n} = 0$$

$$A_2 = A_{2,0} + A_{2,1}x + A_{2,2}x^2 + \dots$$

$$A_1 = 3x$$

$$A_3 = A_{3,0} + A_{3,1}x + A_{3,2}x^2 + A_{3,3}x^3 + \dots$$

$$A_2 = 3x^2$$

$$A_{1+n} = 0$$

$$A_4 = A_{4,0} + A_{4,1}x + A_{4,2}x^2 + A_{4,3}x^3 + A_{4,4}x^4 + \dots$$

5)

3)

$$A_0 = A_{0,0} + A_{0,1}x + A_{0,2}x^2 + \dots + A_{0,n-1}x^{n-1} + A_{0,n}x^n + \dots$$

$$A_1 = A_{1,0} + A_{1,1}x + A_{1,2}x^2 + \dots + A_{1,n-1}x^{n-1} + A_{1,n}x^n + \dots$$

reduce

$$A_2 = A_{2,0} + A_{2,1}x + A_{2,2}x^2 + \dots + A_{2,n-1}x^{n-1} + A_{2,n}x^n + \dots$$

$$A_3 = A_{3,0} + A_{3,1}x + A_{3,2}x^2 + A_{3,3}x^3 + \dots + A_{3,n-1}x^{n-1} + A_{3,n}x^n + \dots + A_{3,2n-1}x^{2n-1} + \dots$$

$$A_4 = A_{4,0} + A_{4,1}x + A_{4,2}x^2 + A_{4,3}x^3 + A_{4,4}x^4 + \dots + A_{4,n-1}x^{n-1} + A_{4,n}x^n + \dots + A_{4,2n-1}x^{2n-1} + \dots$$

$$A_5 = A_{5,0} + A_{5,1}x + A_{5,2}x^2 + A_{5,3}x^3 + A_{5,4}x^4 + A_{5,5}x^5 + \dots + A_{5,n-1}x^{n-1} + A_{5,n}x^n + \dots + A_{5,2n-1}x^{2n-1} + \dots$$

5)