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MECHATRONICS ENG.
16/ENG05/001

Assignment 2

1.)

$y = e^{x^2+x}$

$\therefore \frac{dy}{dx} = (2x+1)e^{x^2+x}$

$y' = (2x+1)e^{x^2+x}$

$\frac{d^2y}{dx^2} = u\frac{dv}{dx} + v\frac{du}{dx}$

$u = (2x+1)$ $\quad$ $v = e^{x^2+x}$

$\frac{du}{dx} = 2$ $\quad$ $\frac{dv}{dx} = (2x+1)e^{x^2+x}$

$\frac{d^2y}{dx^2} = (2x+1)\left[(2x+1)e^{x^2+x}\right] + 2 \times e^{x^2+x}$

~~$\frac{d^2y}{dx^2} =$~~ recall $y' = (2x+1)e^{x^2+x}$

$y = e^{x^2+x}$

$\therefore \frac{d^2y}{dx^2} = (2x+1)y' + 2y$

$\therefore y'' = y'(2x+1) + 2y$

$\therefore y'' - y'(2x+1) 1 - 2y = 0$

n^{th} derivative of y'' : $y^{(n+2)}$

n^{th} derivative of $y'(2x+1)$:

$y'(2x+1) : \sum_{r=0}^{n} {}^nC_r \, u^{(n-r)} v^r$

$\therefore \; y'(2x+1) : {}^nC_0 \, y^{(n+1)}(2x+1) + {}^nC_1 \, y^{(n)}(2) + 0$

$\therefore \; y'(2x+1) : (2x+1) y^{(n+1)} + 2n y^n$

$2y : 2y^n$

$\therefore \; y^{(n+2)} = (2x+1) y^{(n+1)} + 2n y^n + 2y^n$

$y^{(n+2)} = (2x+1) y^{(n+1)} + 2(n+1) y^n$ //

2) i) $y = x^3 e^{4x}$

using Leibnitz theorem

$y^n = \sum_{r=0}^{n} u^{(n-r)} v^{(r)}$

$u = e^{4x}$

$u^n = 4^n e^{4x}$

$V = x^3$

$V' = 3x^2$

$V'' = 6x$

$V''' = 6$

$V'''' = 0$

$\therefore \; y^n = {}^nC_0 \, u^{n-0} v^0 + {}^nC_1 \, u^{(n-1)} v' + {}^nC_2 \, u^{(n-2)} v'' + {}^nC_3 \, u^{(n-3)} v''' + 0$

$y^n = 4^n e^{4x} x^3 + n \, 4^{(n-1)} e^{4x} \, 3x^2 + \frac{n(n-1)(n-2)!}{(n-2)! \, 2!} 4^{(n-2)} e^{4x} . 6x$

$+ \frac{n(n-1)(n-2)(n-3)!}{(n-3)! \, 3!} 4^{(n-3)} e^{4x} . 6 + 0$

$y^n = 4^n e^{4x} x^3 + n 4^{(n-1)} e^{4x} 3x^2 + n(n-1) 4^{(n-2)} e^{4x} \cdot 3x$
$+ n(n-1)(n-2) 4^{(n-3)} e^{4x}$

$y^5 = 4^5 e^{4x} x^3 + 5 4^{(4)} e^{4x} 3x^2 + 5(4) 4^3 e^{4x} 3x$
$+ 5(4)(3) 4^2 e^{4x}$

$y^5 = 1024 e^{4x} x^3 + 3840 x^2 e^{4x} + 3840 x e^{4x} + 960 e^{4x}$

~~iii)~~ ~~x~~

ii) $x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + y = 0$

$x^2 y'' + x y' + y = 0$

Let A be the n^{th} derivative of $x^2 y''$

$A = {}^nC_0 y^{n+2} x^2 + {}^nC_1 y^{n+1} 2x + {}^nC_2 y^n 2 + 0$

$A = y^{(n+2)} x^2 + n y^{(n+1)} 2x + n(n-1) y^n$

Let B be the n^{th} derivative of xy'
$B = {}^nC_0 y^{(n+1)} x + {}^nC_1 y^n \cdot 1 + 0 = y^{(n+1)} x + n y^n$

let C be the n^{th} derivative of y
$C = y^{(n)}$

$\therefore A + B + C = 0$

$\therefore y^{(n+2)} x^2 + n y^{(n+1)} 2x + n(n-1) y^n + x y^{(n+1)} + n y^n + y^n = 0$

$\therefore x^2 y^{(n+2)} + 2xn y^{(n+1)} + x y^{(n+1)} + n(n-1) y^n + n y^n + y^n = 0$

$\therefore \quad x^2 y^{(n+2)} + (2n+1)x y^{(n+1)} + (n^2 - n + n + 1) y^{(n)} = 0$

$\therefore \quad x^2 y^{(n+2)} + (2n+1)x y^{(n+1)} + (n^2+1) y^{(n)} = 0$