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Chemical Engineering ENG 381

$y = e^{x^2+x}$

$y' = (2x+1)e^{x^2+x}$

$u = 2x+1$ $\quad$ $v = e^{x^2+x}$

$\frac{dy}{dx} = 2$ $\quad$ $\frac{dv}{dx} = (2x+1)e^{x^2+x}$

$y'' = u\frac{dv}{dx} + v\frac{du}{dx}$

$= (2x+1)(2x+1)e^{x^2+x} + e^{x^2+x}(2)$

$y'' = y'(2x+1) + 2y$

From Leibinitz Theorem

$y'' - y'(2x+1) - 2y = 0$

$\Downarrow$ $\quad$ $\Downarrow$ $\quad$ $\Downarrow$

W_1 $\quad$ W_2 $\quad$ W_3

$W_1 = y''$, $W_2 = -y'(2x+1)$, $W_3 = -2y$

$W_1 = y''$

$U^0 = y''$, $U' = y'''$

$V^0 = 1$, $V' = 0$

$U^n = y^{n+2}$

$W_1^n = {}^nC_0\, U^{n-0} V^0 + \dots$

$W_1 = y^{n+2}$

$W_2 = y'(2x+1)$

$U^0 = y'$ $\quad$ $U' = y''$

$V^0 = 2x+1$, $V' = 2$

$W_2^n = {}^nC_0 U^{n-0} V^0 + {}^nC_1 U^{n-1} V^1$

$= y^{n+1}(2x+1) + n y^n 2.$

$W_3 = 2y$

$U^0 = y \quad U^1 = y^1$

$V^0 = 2, \quad V^1 = 0$

$U^n = y^n$

$W_3 = {}^nC_0 U^{n-0} V$

$W_3^n = 2y^n$

$\therefore W_1 + W_2 + W_3.$

$y^{n+2} - y^{n+1}(2x+1) - 2n y^n - 2y^n = 0.$

$y^{n+2} - y^{n+1}(2x+1) - 2y^n(n+1) = 0$

$y^{n+2} = y^{n+1}(2x+1) + 2y^n(n+1).$

2) $y = x^3 e^{4x}$

$V^0 = x^3, \quad V' = 3x^2, \quad V^2 = 6x, \quad V^3 = 6$

$U^0 = e^{4x}, \quad U' = 4e^{4x}, \quad U^2 = 16e^{4x}, \quad U^3 = 64e^{4x}$

$U^n = 4^n e^{4x}$

$y^n = {}^nC_0 U^{n-0} V^0 + {}^nC_1 U^{n-1} V' + {}^nC_2 U^{n-2} V^2 + {}^nC_3 U^{n-3} V^3$

$= U^n V^0 + n U^{n-1} V' + \frac{n(n-1)}{2!} U^{n-2} V^2 + \frac{n(n-1)(n-2)}{3!} U^{n-3} V^3$

$= 4^n e^{4x} \cdot x^3 + n 4^{n-1} e^{4x} \cdot 3x^2 + \frac{n(n-1) 4^{n-2} e^{4x}}{2!} 6x + \frac{n(n-1)(n-2) 4^{n-2} e^{4x} \times 6}{3 \times 2}$

$= 4^n e^{4x} \cdot x^3 + n 4^{n-1} e^{4x} \cdot 3x^2 + n(n-1) 4^{n-2} e^{4x} \cdot 3x + n(n-1)(n-2) 4^{n-2} e^{4x}$

$= 4^{n-3} e^{4x} (4^3 x^3 + n 4^2 \cdot 3x^2 + n(n-1) 4 \cdot 3x + n(n-1)(n-2))$

$= 4^{n-3} e^{4x} (64x^3 + (5 \times 48) x^2 + 12 \times 5(5+1) x + 5(5-1)(5-2))$

$y^5 = 4^{5-3} e^{4x} (64x^3 + (5 \times 48) x^2 + 12 \times 5 (5-1) x + 5(5-1)(5-2))$

$y^5 = 16 e^{4x} (64x^3 + 240x^2 + 240x + 60)$

ii $x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + y = 0$

$\underbrace{x^2 y''}_{W_1} + \underbrace{x y'}_{W_2} + \underbrace{y}_{W_3} = 0$

$W_1 = x^2 y''$

$U^0 = y'', \quad U' = y''', \quad U^2 = y^{iv}$

$V^0 = x^2, \quad V' = 2x, \quad V^2 = 2$

$U^n = y^{n+2}$

$W_2 = x y'$

$U^0 = y'$, $U' = y''$, $U^2 = y'''$

$V^0 = x$, $V' = 1$, $V^2 = 0$

$U^n = y^{n+1}$

$W^3 = y$

$U^0 = y$ $U' = y'$

$V^0 = 1$, $V' = 0$

$U^n = y^n$

$W_1^n = {}^nC_0 U^{n-0} V^0 + {}^nC_1 U^{n-1} V' + {}^nC_2 U^{n-2} V^2$

$= U^n V + n U^{n-1} V' + \frac{n(n-1)}{2!} U^{n-2} V^2$

$= y^{n+2} V + n y^{n+1} 2x + \frac{n(n-1)}{\cancel{2}} y^n 2$

$= y^{n+2} V + n y^{n+1} 2x + n(n-1) y^n$

$W_2^n = {}^nC_0 U^{n-0} V^0 + {}^nC_1 U^{n-1} V' + {}^nC_2 U^{n-2} V^2 + {}^nC_3 U^{n-3} V^3$

$= U^n V^0 + n U^{n-1} V' + \frac{n(n-1)}{2!} U^{n-2} . 0$

$= y^{n+1} . x + n y^n . 1 + 0$

$W_3^n = {}^nC_0 U^{n-0} V^0 + {}^nC_1 U^{n-1} V' + 0$

$= U^n V^0 + 0$

$= y^n$

$\therefore W_1 + W_2 + W_3 = 0$

$x^2 y^{n+2} + n 2x y^{n+1} + n(n-1) y^n + y^{n+1} + n y^n + y^n = 0$

$x^2 y^{n+2} + x y^{n+1} (2n+1) + y^n (n^2 - n + n + 1) = 0$

$x^2 y^{n+2} + (2n+1) x y^{n+1} + (n^2+1) y^n = 0$