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16/ENG07/003

1 If $y = e^{x^2+x}$, show that
$y'' = y'(2x+1) + 2y$ and hence prove that $y^{(n+2)} = (2x+1)y^{(n+1)} + 2(n+1)y^n$

Soln

$y = e^{x^2+x}$

$y' = (2x+1)e^{x^2+x}$

$u = 2x+1$; $u' = 2$

$v = e^{x^2+x}$; $v' = (2x+1)e^{x^2+x}$

$y'' = uv' + vu'$

$y'' = (2x+1)\left[(2x+1)e^{x^2+x}\right] + e^{x^2+x} \cdot 2$

but $y' = [2x+1]e^{x^2+x}$ & $y = e^{x^2+x}$

$\therefore$ $y'' = y'(2x+1) + 2y$

$y'' - y'(2x+1) - 2y = 0$

Let $A = y''$

$u = u''$, $u^n = u^{n+2}$

$v = 1$; $v' = 0$

$y^n = u^n v + nv^{n-1}v'$

$y^n = y^{n+2} + ny^{n+1} \cdot 0$

$y^n = y^{n+2} \Rightarrow A$

Let $B = u'(2x+1)$

$u = u'$; $u^n = y^{n+1}$

$v = 2x+1$; $v' = 2$; $v'' = 0$

$y^n = y^{n+1}(2x+1) + ny^n \cdot 2$

$y^n = y^{n+1}(2x+1) + 2ny^n \Rightarrow B'$

Let $C = 2y$

$u = y$; $u^n = y^n$

$v = 2$; $v' = 0$

$y^n = 2y^n$; ny^{n-1} 0

$y^n = 2y^n \Rightarrow C'$

$A - B - C = 0$ 1

$A' - B' - C' = 0$

$y^{n+2} - [y^{n+1}(2x+1) + 2ny^n] - 2y^n = 0$

$y^{n+2} = y^{n+1}(2x+1) + 2ny^n + 2y^n$

$y^{n+1} = y^{n+1}(2x+1) + y^n(2n+2)$

$\therefore y^{n+1} = (2x+1)y^{n+1} + 2(n+1)y^n$

2 Using the leibnitz theorem, given that

i $y = x^3 e^{4x}$, determine y^5

ii $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$, Show that

$x^2 y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2+1)y^n = 0$

Soln

$u = e^{4x}$; $u^n = 4^n e^{4x}$

$v = x^3$; $v' = 3x^2$, $v'' = 6x$; $v''' = 6$, $v^{iv} = 0$

$$y^n = 4^n e^{4x} \cdot x^3 + n4^{n-1}e^{4x}(3x^2) + \frac{n(n-1)4^{n-2}e^{4x} \cdot 6x}{2!}$$

$$+ \frac{n(n-1)(n-2)4^{n-3}e^{4x} \cdot 6}{3!} + 0$$

$y^n = 4^n e^{4x} \cdot x^3 + 3x^2 \cdot n \cdot 4^{n-1} e^{4x} + 3x(n^2-n)4^{n-2}e^{4x}$

$+ n(n^2-3n+2)4^{n-3}e^{4x}$

$y^n = 4^n e^{4x} x^2 + 3x^2 \cdot n \cdot 4^{n-1} e^{4n-1} e^{4x} + 3x(n^2-n)4^{n-2}e^{4x}$

$+ (n^3-3n^2+2n)4^{n-3}e^{4x}$

$y^5 = 1024 e^{4x} x^2 + 3 \cdot 256 \cdot 5x^2 e^{4x} + 3 \cdot (5^2-5) \cdot 4^{n-2} e^{4x} \cdot x$

$+ (5^3 - 3 \cdot 5^2 + 2 \cdot 5) \cdot 4^{n-3} e^{4x}$

$y^5 = 1024x^2 e^{4x} + 3840x^2 e^{4x} + 3840xe^{4x} + 960e^{4x}$

$$y^5 = 64e^{4x}x^3\left[16 + \frac{60}{x} + \frac{60}{x^2} + \frac{15}{x^3}\right]$$

11 $x^2y'' + xy' + y = 0$

$$y^n = u^n v + nu^{n-1}v' + \frac{n(n-1)u^{n-2}v''}{2!} + \frac{n(n-1)(n-2)\cdot u^{n-3}v'''}{3!} + \dots$$

$y = uv$

Let $A = x^2y''$

$u = y''$; $u^n = y^{n+2}$

$v = x^2$; $v' = 2x$; $v'' = 2$; $v''' = 0$

$$y^n = y^{n+2}x^2 + ny^{n+1}\cdot 2x + \frac{n(n-1)y^n \times 2}{2} + \frac{n(n-1)(n-2)y^{n-1}}{3!} \times 0$$

$A' = y^{n+2}x^2 + ny^{n+1}\cdot 2x + n(n-1)y^n$

Let $B = xy'$

$u = y'$; $u^n = y^{n+1}$

$v = x$; $v' = 1$; $v'' = 0$

$$y^n = y^{n+1}\cdot x + ny^{n-1} + \frac{n(n-1)}{2} \times u^{n-1} \times 0$$

$B' \Rightarrow y^n = y^{n+1}\cdot x + ny^n$

let $C = y$

$C' \Rightarrow y^n = y^n$

$A + B + C = 0$

$A' + B' + C' = 0$

$y^{n+2}x^2 + ny^{n+1}\cdot 2x + n(n-1)y^n + y^{n+1}x + ny^n + y^n = 0$

$y^{n+2}x^2 + y^{n+1}x(2n+1) + y^n(n^2 - n + n + 1) = 0$

$y^{n+2}x^2 + y^{n+1}x(2n+1) + y^n(n^2+1) = 0$