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16/ENG07/008
Petroleum Engineering
ENG 381 Assignment 2

1. $y = e^{x^2+x}$

let $u = x^2+x$, $y = e^u$

$\frac{dy}{du} = e^u$ $\quad \frac{du}{dx} = 2x+1$

$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$

$\frac{dy}{dx} = (2x+1) \cdot e^u$

$\frac{dy}{dx} = (2x+1) \cdot e^u$

$y' = (2x+1) \cdot e^u$ $\quad y' = (2x+1)\, e^{x^2+x}$

$\frac{d^2y}{dx^2} = \frac{d}{dx}\left[(2x+1) \cdot e^{x^2+x}\right]$

using product rule

$u = 2x+1 \quad v = e^{x^2+x}$

$\frac{du}{dx} = 2$, $\frac{dv}{dx} = (2x+1) \cdot e^{x^2+x}$

$\frac{d^2y}{dx^2} = v\frac{du}{dx} + u\frac{dv}{dx}$

$\frac{d^2y}{dx^2} = (2x+1) \cdot (2x+1) \cdot e^{x^2+x} + e^{x^2+x} \cdot 2$

recall, $y' = (2x+1) \cdot e^{x^2+x}$, $y = e^{x^2+x}$

$\frac{d^2y}{dx^2} = (2x+1) \cdot (2x+1) \cdot e^{x^2+x} + 2e^{x^2+x}$

$y'' = (2x+1) \cdot y' + 2y$

Hence, $y'' = y'(2x+1) + 2y$

for $w_1(n)$ $\quad u = y^{(2)} \quad v = 1$

$u^{(n)} = y^{(n+2)} \quad v^{(1)} = 0$

From leibniz

for w_2^n

$u = y^{(1)}$ $\quad v = 2x+1$

$u^{(n)} = y^{(n+1)}$ $\quad v^{(1)} = 2$

$u^{(n-1)} = y^{(n)}$ $\quad v^{(2)} = 0$

for w_3^n

$u = y$ $\quad v = 2$

$u^{(n)} = y^{(n)}$ $\quad v^{(1)} = 0$

$$y^{(n+2)} = y^{(n+1)}(2x+1) + n\,2y^{(n)} + 2y^{(n)}$$

$$y^{(n+2)} = y^{(n+1)}(2x+1) + 2y^{(n)}(n+1)$$

Hence, $y^{(n+2)} = y^{(n+1)}(2x+1) + 2(n+1)\,y^{(n)}$

2. Using Leibniz theorem

i, $y = x^3 e^{4x}$, determine $y^{(5)}$

$u = e^{4x}$ $\quad v = x^3$

$u^{(n)} = 4^{(n)} e^{4x}$ $\quad v^{(1)} = 3x^2$

$u^{(n-1)} = 4^{(n-1)} e^{4x}$ $\quad v^{(2)} = 6x$

$u^{(n-2)} = 4^{(n-2)} e^{4x}$ $\quad v^{(3)} = 6$

$u^{(n-3)} = 4^{(n-3)} e^{4x}$ $\quad v^{(4)} = 0$

$$y^{(n)} = u^{(n)}v + n u^{(n-1)} v^{(1)} + \frac{n(n-1)}{2!} u^{(n-2)} v^{(2)} +$$

$$\frac{n(n-1)(n-2)}{3!} u^{(n-3)} v^{(3)} + \cancel{\frac{n(n-1)(n-2)(n-3)}{4!} u^{(n-4)} v^{(4)}} \; 0$$

$$y^{(5)} = 4^5 e^{4x} \cdot x^3 + 5 \cdot 4^4 e^{4x} \cdot 3x^2 + \frac{5 \cdot 4}{2!} 4^3 e^{4x} \cdot 6x +$$

$$+ \frac{5 \cdot 4 \cdot 3}{3!} 4^2 e^{4x} \cdot 6$$

$$y^{(5)} = 1024 x^3 e^{4x} + 3840 x^2 e^{4x} + 3840 x e^{4x} + 960 e^{4x}$$

$$y^{(5)} = 64 e^{4x} (16x^3 + 60x^2 + 60x + 15)$$

ii, $x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + y = 0$

$x^2 y'' + x y' + y = 0$

for $w_1^{(n)}$

$u = y^{(2)}$ $\quad v = x^2$

$u^{(n)} = y^{(n+2)}$ $\quad v^{(1)} = 2x$

$u^{(n-1)} = y^{(n+1)}$ $\quad v^{(2)} = 2$

$u^{(n-2)} = y^{(n)}$ $\quad v^{(3)} = 0$

for $w_2^{(n)}$

$u = y^{(1)}$ $\quad v = x$

$u^{(n)} = y^{(n+1)}$ $\quad v^{(1)} = 1$

$u^{(n-1)} = y^{(n)}$ $\quad v^{(2)} = 0$

for $w_3^{(n)}$

$u = y$ $\quad v = 1$

$u^n = y^n$ $\quad v = 0$

$$w_1^{(n)} = y^{(n+2)} x^2 + n\, y^{(n+1)}\, 2x + \frac{n(n-1)}{\cancel{2!}}\, y^{(n)}\, \cancel{2}$$

$$w_2^{(n)} = y^{(n+1)} x + n\, y^{(n)}$$

$$w_3^{(n)} = y^n$$

$$w_1^{(n)} + w_2^{(n)} + w_3^{(n)} = 0$$

$$0 = y^{(n+2)} x^2 + n\, y^{(n+1)}\, 2x + n(n-1)\, y^{(n)} + y^{(n+1)} x + n\, y^{(n)} + y^{(n)}$$

$$0 = y^{(n+2)} x^2 + x\, y^{(n+1)} (2n+1) + y^{(n)} (n^2 - n + n + 1)$$

$$0 = y^{(n+2)} x^2 + x\, y^{(n+1)} (2n+1) + y^{(n)} (n^2 + 1)$$