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QU 16/ENG04/048

ENG381 - Engineering Maths.

1.) $y = e^{(x^2+x)}$ $y = e^{p}$

let $p = x^2+x$

$\frac{dy}{dx} = \frac{dy}{dp} \cdot \frac{dp}{dx}$; $\frac{dy}{dp} = e^{p}$ $\frac{dp}{dx}$ $2x+1$

$\frac{dy}{dx} = e^{p} \cdot 2x+1 = (2x+1)e^{p}$

$\frac{dy}{dx} = (2x+1)e^{x^2+x}$

$\frac{dy}{dx} = (2x+1) \cdot e^{x^2+x}$

$y^{+} = \frac{d(dy/dx)}{dx} = d^2y/dx^2 = y''$

using product rule.

y'' let $u = 2x+1$

$v = e^{x^2+x}$

$\frac{d(dy/dx)}{dx} = u\, dv/dx + v\, du/dx$

recall $\frac{dy}{dx}$ $\frac{d(e^{x^2+x})}{dx} = (2x+1) \cdot e^{x^2+x}$

$\frac{dv}{dx} = \frac{d(e^{x^2+x})}{dx} = (2x+1) \cdot e^{x^2+x}$

$\frac{du}{dx} = 2.$

$\frac{d(dy/dx)}{dx^2} = \frac{d^2y}{dx^2} = y'' = u\frac{dv}{dx} + v\frac{du}{dx}$

$y'' = (2x+1) \cdot (2x+1)e^{x^2+x} + e^{x^2+x}(2)$

recall $y = e^{x^2+x}$

$dy/dx = y' = (2x+1) \cdot e^{x^2+x}.$

$y'' = (2x+1) \cdot y' + y \cdot 2$

$y'' = (2x+1)y' + 2y$ //.

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y^{n+2}

from leibnitz.

$y'' = y'(2x+1) + 2y$

$w_1 \rightarrow \quad v = 1 \qquad u^n = y^{n+2}$

$v^{(1)} = 0 \qquad u^{(n-1)} = y^{(n+1)}$

$w_1 = {}^nC_0 \cdot u^n \cdot v + {}^nC_1 u^{n-1} \cdot v'$

$= y^{n+2} \cdot 1 + n \cdot 0 \cdot y^{(n+1)} = y^{n+2}.$

$w_2 \rightarrow \quad v = 2x+1 \qquad u^n = y^{n+1}$

$v' = 2 \qquad u^{n-1} = y^n$

$v^2 = 0 \qquad u^{n-2} = y^{n-1}$

$w_2 = {}^nC_0 u^n \cdot v + {}^nC_1 \cdot u^{n-1} \cdot v' + {}^nC_2 \cdot u^{n-2} \cdot v^2$

$= y^{n+1} \cdot 2x+1 + n \cdot y^n \cdot 2 + \frac{n \cdot (n-1)}{2!} \cdot 0 + y^{n-1}$

$= y^{n+1} \cdot 2x+1 + 2ny^n$

$w_3 \rightarrow \quad v = 2 \qquad u^n = y^n$

$v' = 0 \qquad u^{n-1} = y^{n-1}$

$w_3 = {}^nC_0 u^n \cdot v + {}^nC_1 u^{n-1} \cdot v'$

$= y^n \cdot 2 + n \cdot 0 \cdot y^{n-1} = 2y^n$

$w_1 = w_2 + w_3$

$= y^n$

$y^{n+2} = y^{n+1} \cdot 2x+1 + 2ny^n + 2y^n$

$y^{n+2} = y^{(n+1)}(2x+1) + 2y^n(n+1)$

$y^{n+2} = y^{n+1}(2x+1) + 2 \cdot (n+1) y^n$ //

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16/ENG04/043
Electrical Electronics.
ENG 381.

Engineering Mathematics.

2.) Using Leibnitz theorem.

$y = x^3 e^{4x}$ find y^5.

$V = x^3$ $\quad U = e^{4x}$

$V^{(1)} = 3x^2$ $\quad U^n = 4^n \cdot e^{4x}$

$V^{(2)} = 6x$ $\quad U^{n-1} = 4^{(n-1)} \cdot e^{4x}$

$V^{(3)} = 6$ $\quad U^{(n-2)} = 4^{(n-2)} \cdot e^{4x}$

$U^{(n-3)} = 4^{(n-3)} \cdot e^{4x}$

$y^n = {}^nC_0 \cdot U^n \cdot V^{(0)} + {}^nC_1 + U^{(n-1)} \cdot V^{(1)} + {}^nC_2 + U^{(n-2)} \cdot V^2 + {}^nC_3 + U^{(n-3)} \cdot V^3$

$\cancel{y^n = 1 \cdot 4^n \cdot e^{4x} \cdot x^3 + n \cdot 4^{(n-1)} \cdot e^{4x} + \frac{n(n-1)}{2!} \cdot 4^{n-2}}$

$y^n = 1 \cdot 4^n e^{4x} \cdot x^3 + 3x^2 \cdot n \cdot 4^{(n-1)} e^{4x} + \frac{n \cdot (n-1)}{2!} \cdot 4^{(n-2)} \cdot 6x \cdot e^{4x}$

$+ \frac{n(n-1) \cdot (n-2)}{3!} \cdot 4^{(n-3)} \cdot e^{4x} \cdot 6$

$y^n = 4^n e^{4x} \cdot x^3 + 3nx^2 \cdot 4^{(n-1)} \cdot e^{4x} + n \cdot (n-1) \cdot 4^{(n-2)} e^{4x} \cdot 3x$
$+ n(n-1)(n-2) \cdot 4^{(n-3)} e^{4x}$

$y^n = e^{4x} \left(4^n \cdot x^3 + 3nx^2 \cdot 4^{(n-1)} + n \cdot (n-1) \cdot 4^{(n-2)} \cdot 3x + n(n-1)(n-2) \cdot 4^{(n-3)}\right)$

$y^5 = e^{4x} (4^5 \cdot x^3 + 3 \cdot 5x^2 \cdot 4^4 + 5 \times 4 \times 4^3 \cdot 3x + 5 \times 4 \cdot 3 \cdot 4^2$

$= e^{4x} (1024x^3 + 3840x^2 + 3840x + 960)$

$= 64e^{4x} (16x^3 + 60x^2 + 60x + 15)$

Show using Leibnitz theorem

$$x^2 \cdot y^{(n+2)} + (2n+1)xy^{n+1} + y^n(n^2+1) = 0$$

4.

i.) $x^2 \dfrac{d^2y}{dx^2} + x\dfrac{dy}{dx} + y = 0$

$x^2 y'' + xy' + y = 0$

$w_1 \rightarrow$ $V = x^2$ $\quad u^n = y^{n+2}$

$V' = 2x$ $\quad u^{(n-1)} = y^{n+1}$

$V^{(2)} = 2$ $\quad u^{(n-2)} = y^n$

$w_1 = {}^nC_0 \cdot u^n \cdot V^0 + {}^nC_1 \cdot y^{(n-1)} \cdot V' + {}^nC_2 \cdot u^{(n-2)} \cdot V^2$

$= y^{n+2} \cdot x^2 + n \cdot y^{n+1} + 2x + \dfrac{n \cdot (n-1)}{2!} \cdot y^n + 2.$

$w_1 = y^{n+2} \cdot x^2 + 2nx y^{n+1} + \dfrac{\cancel{2}n(n-1)y^n}{\cancel{2}}$

$w_2 \rightarrow$ $V = x$ $\quad u^n = y^{n+1}$

$V' = 1$ $\quad u^{n+1} = y^n$

$V^2 = 0$ $\quad u^{n-2} = y^{n-1}$

$w_2 = {}^nC_0 \cdot u^n \cdot V^0 + {}^nC_1 \cdot u^{(n-1)} \cdot V' - {}^nC_2 \cdot u^{(n-2)} \cdot V^2$

$= y^{n+1} \cdot x + n \cdot y^n \cdot 1 + 0 \quad = xy^{n+1} + ny^n$

w_3 $\quad V = 1$ $\quad u^n = y^n$

$V' = 0$ $\quad u^{n-1} = y^{n-1}$

$w_3 \rightarrow {}^nC_0 \cdot u^n \cdot V^0 + {}^nC_1 \cdot u^{(n-1)} \cdot V'$

$= 1 \cdot y^n + 0 \quad = y^n$

$w_1 + w_2 + w_3 = 0$

$0 = x^2 \cdot y^{n+2} + 2nxy^{n+1} + n \cdot (n-1)y^n + xy^{n+1} + ny^n + y^n$

$0 = x^2 \cdot y^{n+2} + 2nxy^{n+1} + xy^{n+1} + n(n-1)y^n + ny^n + y^n$

$0 = x^2 \cdot y^{n+2} + xy^{n+1}(2n+1) + y^n(n^2 - n + n + 1)$

$0 = x^2 \cdot y^{n+2} + (2n+1)(xy^{n+1}) + y^n(n^2+1)$

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