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COURSE CODE: ENG 381

DEPARTMENT: Chemical

1) $y = e^{x^2+x}$

$y' = (2x+1)e^{x^2+x}$

$u = 2x+1$ $\quad$ $v = e^{x^2+x}$

$\frac{du}{dx} = 2$ $\quad$ $\frac{dv}{dx} = (2x+1)e^{x^2+x}$

$y'' = u\frac{dv}{dx} + v\frac{du}{dx}$

$y'' = (2x+1)(2x+1)e^{x^2+x} + e^{x^2+x}(2)$

$y'' = y'(2x+1) + 2y$

From Leibnitz Theorem

$y'' - y'(2x+1) - 2y = 0$

$W_1 = y''$

$u = y''$ $\quad$ $V = 1$

$u' = y'''$ $\quad$ $V' = 0$

$u^n = y^{n+2}$

$= \cancel{u^n v^0 + nu^n v' + \frac{nu^{n-1}}{2}}$

$= u^n v^0 + nu^{(n-1)}v'$

$= y^{n+2} \cdot 1 + ny^{n+3} \cdot 0$

$W_1 = y^{n+2}$

$W_2 = y'(2x+1)$

$u = y'$ $\quad$ $v = 2x+1$

$u' = y''$ $\quad$ $\cancel{v' = 2x}$ $\quad$ $v' = 2$ $\quad$ $v'' = 0$

$= u^n = y^{n+1}$

$= u^n v^0 + nu^{(n-1)}v'$

$= y^{n+1}(2x+1) + ny^{\cancel{n+2}}(2)$

$W_2 = y^{n+1}(2x+1) + ny^n \cdot 2$

$W_3 = 2y$

$\cancel{u=2x}$ $u = y$ $\qquad v = 2$

$u' = y'$ $\qquad v' = 0$

$u^n = y^n$

$\cancel{=}\ u^n \cdot v^0 + n u^{n-1} \cdot v'$

$= y^n \cdot 2 + n u^{n-1} \cdot 0$

$W_3 = 2y^n$

$y'' - y'(2x+1) - 2y = 0$

$y^{n+2} - y^{n+1}(2x+1) - 2ny^n - 2y^n = 0$

$y^{n+2} - y^{n+1}(2x+1) - 2y^n(n+1) = 0$

$y^{n+2} = y^{n+1}(2x+1) + 2y^n(n+1)$

2 $y = x^3 e^{4x}$

$v^0 = x^3 \quad v' = 3x^2, \quad v'' = 6x, \quad v''' = 6$

$u = e^{4x} \quad u' = 4e^{4x}, \quad u'' = 16e^{4x}, \quad u''' = 64e^{4x}$

$u^n = 4^n e^{4x}$

$$y = u^n v^0 + n u^{n-1} \cdot v' + \frac{n(n-1)}{2!} \cdot u^{n-2} \cdot v'' + \frac{n(n-1)(n-2)}{3\times 2} \cdot u^{n-3} \cdot v'''$$

$$y = 4^n e^{4x} \cdot x^3 + n \cdot 4^{n-1} e^{4x} \cdot 3x^2 + \frac{n(n-1)}{2} 4^{n-2} e^{4x} \cdot 6x + \frac{n(n-1)(n-2)}{3\times 2} 4^{n-3} e^{4x} \cdot 6$$

$$y = 4^n e^{4x} \cdot x^3 + n \cdot 4^{n-1} e^{4x} \cdot 3x^2 + n(n-1) 4^{n-2} e^{4x} \cdot 3x + n(n-1)(n-2) \cdot 4^{n-3} e^{4x}$$

$$= 4^{n-3} e^{4x} (4^3 x^3 + n 4^2 \cdot 3x^2 + n(n-1) 4 \cdot 3x + n(n-1)(n-2)]$$

$$= 4^{n-3} e^{4x} (64x^3 + n 48x^2 + 12n(n-1) + n(n-1)(n-2)]$$

$$y^5 = 4^{5-3} e^{4x} (64x^3 + (5\times 48)x^2 + 12\times 5(5-1)x + 5(5-1)(5-2)]$$

$$= 16e^{4x}(64x^3 + 240x^2 + 240x + 60)$$

3 $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$, show that $x^2 y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2+1)y^{(n)} = 0$

$x^2y'' + xy' + y = 0$

$W_1 = x^2y''$

$u^0 = y''$ $\quad v^0 = x^2$

$u' = y'''$ $\quad v' = 2x$

$u'' = y^{iv}$ $\quad v'' = 2$

$u^n = y^{n+2}$

$W_2 = xy'$

$u^0 = y'$ $\quad v = x$

$u' = y''$ $\quad v' = 1$

$u'' = y'''$ $\quad v'' = 0$

$u^n = y^{n+1}$

$W_3 = y$

$u = y$ $\quad v = 1$

$u' = y'$ $\quad v' = 0$

$u'' = y''$

$u^n = y^n$

$$y = u^n v^0 + nu^{n-1}v' + \frac{n(n-1)}{2!}u^{n-2}v'' + \frac{n(n-1)(n-2)}{3!}u^{n-2}v'''$$

$$= y^{n+2}v + ny^{n+1}2x + \frac{n(n-1)}{x}y^n x$$

$$= y^{n+2}v + ny^{n+1}2x + n(n-1)y^n$$

$W_1 = y^n(y^2x^2 + ny\,2x + n(n-1))$

$n-2+n+1$

$$y = u^n v^0 + nu^{(n-1)}v' + \frac{n(n-1)}{2!}u^{n-2}v'' + \frac{n(n-1)(n-2)}{3!}u^{(n-2)}v'''$$

$= y^{n+1}\cdot x + ny^n\cdot 1 + 0$

$W_2 = y^n(xy + n)$

$$y = u^n v^0 + n u^{(n-1)} v^1 + \frac{n(n-1) u^{n-2}}{2!} v'' + \frac{n(n-1)(n-2) u^{n-3}}{3!} v'''$$

$$= u^n v^0 + 0$$

$$= y^n$$

$x^2 y'' + xy' + y = 0$

$w_1 + w_2 + w_3 = 0$

$y^n [y^2 x^2 + n 2xy + n(n-1)] + y^n (xy + n) + y^n = 0$

$x = 0$

$\cancel{y^n n(n-1) + y^n n + y^n = 0}$

$\cancel{n(n-1) + n + 1 = 0}$

$\cancel{y^n = -y^n n(n-1) - n y^n}$

$\cancel{\text{at } n = 1}$

$\cancel{y = -y 0 - y'}$

$\cancel{y = -y'}$

$x^2 y^{n+2} + n 2x y^{n+1} + n(n-1) y^n + x y^{n+1} + n y^n + y^n = 0$

$x^2 y^{n+2} + x y^{n+1} (2n+1) + y^n (n^2 - n + n + 1) = 0$

$x^2 y^{n+2} + (2n+1) x y^{n-1} + (n^2+1) y^n = 0$