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16/ENG01/006
Chemical eng.

If $y = e^{x^2+x}$ show that $y'' = y'(2x+1) + 2y$ prove that $y^{(n+2)} = (2x+1)y^{(n)}$

Soln

$y = e^{x^2+x}$

let $u = x^2 + x$. $y = e^u$

$\frac{du}{dx} = 2x+1$, $\frac{dy}{du} = e^u$

$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx} = (2x+1)e^{x^2+x}$

to y'' = using product rule.

$y'' = u\, v\frac{du}{dx} + u\frac{dv}{dx}$

dv ~~let u~~ $\frac{dv}{dx} = (2x+1)e^{x^2+x}$ $du/dx = 2$.

$y'' = (2x+1)(2x+1)e^{x^2+x} + e^{x^2+x}(2)$

Since $y' = (2x+1)e^{x^2+x}$ and $y = e^{x^2+2}$.

$y'' = y'(2x+1) + 2y$

$y'' - y'(2x+1) + 2y = 0$.

Let $w_1 = y''$.

$u^{(0)} = y''$, $u^{(1)} = y'''$ $u^{(11)} = y^{iv}$.

$u^n = y^{(n+2)}$, $w_1^{(n)} = y^{(n+2)}$

Let $w_2 = y'(2x+1)$

$u^{(0)} = y'$, $u^{(1)} = y''$, $u^{(11)} = y'''$, $u^n = y^{(n+1)}$

$v^{(0)} = 2x+1$, $v^{(1)} = 2$, $v^{(11)} = 0$.

$w_2^{(n)} = y^{(n+1)} \cdot 2x+1 + ny^n 2$.

$= y^{(n+1)}(2x+1) + 2ny^n$

$w_3 = 2y$

$u = y$ $u^n = y^n$

$w_3^{(n)} = y^{(n)}$

$y^{(n+2)} - y^{(n+1)}(2x+1) + 2ny^n) - 2y^n = 0$.

$y^{(n+2)} = y^{(n+1)}(2x+1) + 2ny^n + 2y^n$.

(2) $y = x^3 e^{4x}$ determine $y^{(5)}$.

let $v = x^3$, $U = e^{4x}$

$v^{(0)} = x^3 \quad v' = 3x^2 \quad v^2 = 6x \quad v^3 = 6$

$U^{(0)} = e^{4x} \quad u' = 4e^{4x}, \quad U'' = 16e^{4x}, \quad U''' = 64e^{4x}$

$U^{(n)} = 4^n \cdot e^{4x}$.

$y^{(n)} = 4^n e^{4x} \cdot x^3 + n4^{n-1} e^{4x} \cdot 3x^2 + \frac{n(n-1)}{2!} 4^{n-2} e^{4x} \cdot 6x + \frac{n(n-1)(n-2)}{3!} 6$

$y^n = 4^{(n-3)} e^{4x} [4^3 x^3 + n4^2 3x^2 + n(n-1) 2 \cdot 6x + n(n-1)(n-2)$

$y^n = 4^{(n-3)} e^{4x} [64x^3 + 48nx^2 + 12n(n-1)x + n(n-1)(n-2)]$

$y^5 = 4^2 \cdot e^{4x} [64x^3 + 240x^2 + 240x + 60]$

$y^5 = 16e^{4x} [64x^3 + 240x^2 + 240x + 60]$

$y^5 = 1024x^3 e^{4x} + 3840x^2 e^{4x} + 3840x^2 e^{4x} + 960e^{4x}$

11) $x^2 y'' + xy' + y = 0$. Show

Let $W_1 = x^2 y'' \quad W_2 = xy'. \quad W_3 = y. = 0$

$W_1^{(n)} = x^2 y''$

$V^{(0)} = x^2, \quad V' = 2x \quad V^2 = 2.$

$U^{(0)} = y'', \quad U' = y''', \quad U'' = y^{iv}. \quad U^n = y^{(n+2)}$

$W_1^{(n)} = y^{(n+2)} \cdot x^2 + ny^{n+1} \cdot 2x + \frac{n(n-1)}{2} y^{(n)} \cdot 2.$

$W_1^{(n)} = x^2 y^{(n+2)} + 2xny^{(n+1)} + n(n-1)y^n.$

$W_2^{(n)} = xy'$

$V^0 = x \quad V' = 1$

$U^{(0)} = y' \quad U^{(1)} = y'' \quad U^n = y^{(n+1)}$

$W_2^{(n)} = y^{(n+1)} \cdot x + ny^n \cdot 1$

$W_2^{(n)} = xy^{(n+1)} + ny^n$.

$W_3 = y.$

$U^{(0)} = y \quad U^{(1)} = y' \quad U^n = y^{(n)}$

$V^{(0)} = 1 \quad V^{(1)} = 0.$

$W_3^{(n)} = y^{(n)}$

$W_1^{(n)} + W_2^{(n)} + W_3^{(n)} = 0$

$x^2y^{(n+2)} + 2xny^{(n+1)} + n(n-1)y^n + xy^{(n+1)} + ny^n + y^n = 0.$

$x^2y^{(n+2)} + 2xny^{(n+1)} + xy^{(n+1)} + n(n-1)y^n + ny^n + y^n = 0$

$x^2y^{(n+2)} + xy^{(n+1)}(2n+1) + y^n(n(n-1)+n+1) = 0$

$x^2y^{(n+2)} + xy^{(n+1)}(2n+1) + y^n(n^2-n+n+1) = 0.$

$x^2y^{(n+2)} + xy^{(n+1)}(2n+1) + y^n(n^2+1) = 0$

$x^2y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2+1)y^n = 0$ Qed //

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