

$$y = e^{x^2+x}$$

Show that $y'' = y'(2x+1) + 2y$

~~$\frac{dy}{dx} = e^x$~~

let $x^2+x = u$; $\frac{du}{dx} = (2x+1)$

$$y = e^u \quad \frac{dy}{du} = e^u$$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$= e^u \times (2x+1)$$
$$= (2x+1)e^{x^2+x}$$

$$\frac{d^2y}{dx^2} = y'' = u \frac{dy}{dx} + v \frac{dy}{dx}$$

$$= (2x+1)(2x+1)e^u + e^{x^2+x}(2)$$

Recall! $y = e^{x^2+x}$
 $y' = (2x+1)e^u$

$$y'' = (2x+1)y' + 2y$$



From y^2

$$y^{(u)} = (2x+1)y^{(u-1)} + 2(u-1)y^u$$

$$\therefore y^{(u+2)} = (2x+1)y^{(u+2-1)} + 2(u+2-1)y^u$$

$$y^{(u+2)} = (2x+1)y^{(u+1)} + 2(u+1)y^u$$

$$2) \text{ if } y = x^3 e^{4x}$$

\downarrow
 v

\downarrow
 u

$$\sum_{r=0}^n {}^n C_r u^{(n-r)} v^r$$

$$v = x^3, v^{(1)} = 3x^2, v^{(2)} = 6x, v^{(3)} = 6, v^{(4)} = 0, v^{(5)} = 0$$

$$u = e^{4x}, u^{(1)} = 4e^{4x}, u^{(2)} = 16e^{4x}, u^{(3)} = 64e^{4x}, u^{(4)} = 256e^{4x}, u^{(5)} = 1024e^{4x}$$

$$y^{(5)} = u^{(5)}v + 5u^{(4)}v^{(1)} + {}^5 C_2 u^{(3)}v^{(2)} + {}^5 C_3 u^{(2)}v^{(3)} + {}^5 C_4 u^{(1)}v^{(4)}$$

$$= u^{(5)}v + 5u^{(4)}v^{(1)} + {}^5 C_2 u^{(3)}v^{(2)} + {}^5 C_3 u^{(2)}v^{(3)} + {}^5 C_4 u^{(1)}v^{(4)}$$

$$= 1024e^{4x}x^3 + 5(256e^{4x})(3x^2) + 10(64e^{4x})6x + 10(16e^{4x})6 + 5(4e^{4x})(0)$$

$$= 1024e^{4x}x^3 + 3840e^{4x}x^2 + 3840e^{4x}x + 960e^{4x} + 0$$

$$y^{(5)} = e^{4x}(1024x^3 + 3840x^2 + 3840x + 960)$$

$$3) x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + y = 0 \text{ solve } x^2 y^{(n+2)} + (2n+1)x y^{(n+1)} + (n^2+1)y^{(n)} = 0$$

$$x^2 y'' + x y' + y = 0$$

$$x^2 y'' = -x y' - y$$

$$v = x^2, v^{(1)} = 2x, v^{(2)} = 2$$

$$u = y'', u^{(1)} = y^{(3)}, u^{(2)} = y^{(4)} \quad u^{(n)} = y^{(n+2)}$$

$$\omega_1^{(n)} = \sum_{r=0}^n {}^n C_r u^{(n-r)} v^r$$

$$= u^{(n)}v + n u^{(n-1)}v^{(1)} + {}^n C_2 u^{(n-2)}v^{(2)} = u^{(n)}v + n u^{(n-1)}v^{(1)} + \frac{n(n-1)}{2} u^{(n-2)}v^{(2)}$$

$$= y^{(n+2)}x^2 + n y^{(n+1)}2x + \frac{n(n-1)}{2} y^{(n)}(2)$$

$$\omega_1^{(n)} = y^{(n+2)} x^2 + n y^{(n+1)} 2x + n(n-1) y^n$$

$$2y' = \omega_2$$

$$v = x, v^{(1)} = 1$$

$$y = y^{(1)}, u^{(1)} = y^{(2)} \quad u^n = y^{(n+1)}$$

$$\begin{aligned} \omega_2 &= u^{(1)} v + n u^{(n-1)} v^{(1)} \\ &= y^{(2)} x + n y^{(1)} \end{aligned}$$

$$\omega_3 = y'$$

$$u = 1, x^{(1)} = 1$$

$$u = y' \quad u^{(1)} = y^{(2)}$$

$$\omega_3 = u^{(1)} v$$

$$\omega_3 = x y''$$

$$\omega_1^{(1)} + \omega_2^{(1)} + \omega_3^{(1)} = 0$$

$$x^2 y^{(n+2)} + n y^{(n+1)} 2x + u(n-1) y^n + y^{(n+1)} x + n y^{(1)} + y^{(1)} = 0$$

$$x^2 y^{(n+2)} + x(1+2n) y^{(n+1)} + (n(n-1) + n) y^n = 0$$

$$x^2 y^{(n+2)} + (2n+1) x y^{(n+1)} + (n^2 + n) y^{(1)} = 0$$

$$x^2 y^{(n+2)} + (2n+1) x y^{(n+1)} + (n^2 + n) y^{(1)} = 0$$