ENG 381 ASSIGNMENT II

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DEPT: CIVIL ENGINEERING

LEVEL: 300 LEVEL

1 If $y = e^{x^2+x}$,

show that $y'' = y'(2x+1) + 2y$

and hence, prove that

$$y^{(n+2)} = (2x+1)y^{(n+1)} + 2(n+1)y^n$$

Solution

a $y = e^{x^2+x}$

$y' = (2x+1)e^{x^2+x}$

$$y'' = u\frac{dv}{dx} + v\frac{du}{dx}$$

let $u = 2x+1$, $\frac{du}{dx} = 2$

$v = e^{x^2+x}$, $\frac{dv}{dx} = (2x+1)e^{x^2+x}$

$$y'' = (2x+1)(2x+1)e^{x^2+x} + e^{x^2+x}(2)$$

$$y'' = (2x+1)e^{x^2+x}(2x+1) + 2e^{x^2+x}$$

Recall: $y = e^{x^2+x}$, $y' = (2x+1)e^{x^2+x}$

$$\therefore \quad y'' = y'(2x+1) + 2y$$

b $y^n = \sum_{r=0}^{n} {}^nC_r U^{(n-r)} V^r$ [Leibnitz Theorem]

$$y^n = {}^nC_0 U^n V^0 + {}^nC_1 U^{(n-1)} V^{(1)} + {}^nC_2 U^{(n-2)} V^{(2)} + \dots$$

for y'' : $y^{(n+2)}$

for $y'(2x+1)$: y^n ${}^nC_0(y^{(n+1)})(2x+1) + {}^nC_1(y^{(n+1-1)})(2x+1)^{(1)} + 0$

$$y^{(n+1)}(2x+1) + ny^{(n)} \cdot 2$$

for $2y$: $2y^{(n)}$

$\therefore y^{(n+2)} = \cancel{y^{(n+1)}} \{ y^{(n+1)}(2x+1) + n y^{(n)} \cdot 2 + 2y^{(n)}$

$\cancel{= y^{(n+2)} +}$

$y^{(n+2)} = (2x+1) y^{(n+1)} + 2n y^{(n)} + 2y^{(n)}$

$$\underline{\underline{y^{(n+2)} = (2x+1) y^{(n+1)} + 2(n+1) y^{(n)}}}$$

2 Using Leibnitz theorem, given that

ii $y = x^3 e^{4x}$, determine $y^{(5)}$.

Solution

$y^{n} = \sum_{r=0}^{n} {}^{n}C_r U^{(n-r)} V^{(r)}$ [Leibnitz theorem]

let $U = e^{4x}$ and $V = x^3$

$\therefore U^{(1)} = 4e^{4x}$ $\quad V^{(1)} = 3x^2$

$U^{(2)} = 16e^{4x}$ $\quad V^{(2)} = 6x$

$U^{(3)} = 64e^{4x}$ $\quad V^{(3)} = 6$

$U^{(4)} = 256e^{4x}$ $\quad V^{(4)} = 0$

$U^{(5)} = 1024e^{4x}$ $\quad V^{(5)} = 0$

$y^{(5)} = {}^{5}C_0 U^{(5)} V^{(0)} + {}^{5}C_1 U^{(4)} V^{(1)} + {}^{5}C_2 U^{(3)} V^{(2)} + {}^{5}C_3 U^{(2)} V^{(3)} + {}^{5}C_4 U^{(1)} V^{(4)}$
$+ {}^{5}C_5 U^{(0)} V^{(5)}$

$y^{(5)} = (1 \times 1024e^{4x} \times x^3) + (5 \times 256e^{4x} \times 3x^2) + (10 \times 64e^{4x} \times 6x)$
$+ (10 \times 16e^{4x} \times 6) + 0 + 0$

$= \cancel{1024e^{4x}}$

$$\underline{\underline{y^{(5)} = 1024x^3 e^{4x} + 3840x^2 e^{4x} + 3840x e^{4x} + 960 e^{4x}}}$$

2) $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$, Show that $x^2y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2+1)y^{(n)} = 0$

Solution

$y^n = \sum_r^n {}^nC_r U^{(n-r)} V^r$

~~for $x^2y^{(2)}$~~:

$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

$x^2y^{(2)} + xy^{(1)} + y = 0$

for $x^2y^{(2)}$: ${}^nC_0 y^{(n+2)} x^2 + {}^nC_1 y^{(n+2-1)} (x^2)^{(1)} + {}^nC_2 y^{(n+2-2)} (x^2)^{(2)} + 0$

$= y^{(n+2)}x^2 + ny^{(n+1)}\cdot 2x + \frac{n(n-1)}{2!}y^n\cdot 2$

for $xy^{(1)}$: ${}^nC_0 y^{(n+1)}\cdot x + {}^nC_1 y^{(n+1-1)}(x)^{(1)} + 0$

$= y^{(n+1)}x + ny^{(n)}$

for y : ${}^nC_0 y^{(n)}$

$= y^{(n)}$

$\therefore$ $x^2y^{(2)} + xy^{(1)} + y = 0$

becomes : $y^{(n+2)}x^2 + ny^{(n+1)}\cdot 2x + \frac{n(n-1)}{2!}y^n\cdot 2 + y^{(n+1)}x + ny^{(n)} + y^{(n)} = 0$

$x^2y^{(n+2)} + 2xny^{(n+1)} + \frac{\cancel{2}n(n-1)}{\cancel{2}}y^n + xy^{(n+1)} + ny^{(n)} + y^{(n)} = 0$

$x^2y^{(n+2)} + (2n+1)xy^{(n+1)} + (n(n-1) + n + 1)y^{(n)} = 0$

$x^2y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2 - n + n + 1)y^{(n)} = 0$

$x^2y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2+1)y^{(n)} = 0$