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Solution

$$1) y = e^{x^2+x}$$

$$y' = (2x+1)e^{x^2+x}$$

$$u = 2x+1$$

$$\frac{du}{dx} = 2$$

$$v = e^{x^2+x}$$

$$\frac{dv}{dx} = (2x+1)e^{x^2+x}$$

$$y'' = u \frac{dv}{dx} + v \frac{du}{dx}$$
$$= (2x+1)(2x+1)e^{x^2+x} + e^{x^2+x}(2)$$

$$y'' = y'(2x+1) + 2y$$

From Leibnitz theorem

$$y'' - y'(2x+1) - 2y = 0$$

$$w_1 = y''$$

$$v = 1$$

$$u = y'$$

$$v' = 0$$

$$u^n = y^{n+2}$$

$$m_1^n = {}^n C_2 u^{n-0} v^2$$
$$= y^{n+2}$$

$$w_2 = y'(2x+1)$$

$$v' = 2$$

$$u = y'$$

$$v'' = 0$$

$$u^n = y^{n+1}$$

$$v_1^n = {}^n C_0 u^{n-0} v^0 + {}^n C_1 u^{n-1} v^1$$

$$= u^n v + n u^{n-1} v'$$

$$= y^{n+1} (2x+1) + n y^{n-2}$$

$$W_3 = 2y$$

$$u = y, v = 2, v' = 0$$

$$u^n = y^n$$

$$W_3^n = {}^n C_0 u^{n-0} v$$

$$= 2y^n$$

$$y^n - y'(2x+1) - 2y = 0$$

$$y^{n+2} - y^{n+1} (2x+1) - 2ny^n - 2y^n = 0$$

$$y^{n+2} - y^{n+1} (2x+1) - 2y^n (n+1) = 0$$

$$y^{n+2} = y^{n+1} (2x+1) + 2y^n (n+1)$$

$$2) i) y = x^3 e^{4x}$$

$$v^0 = x^3, v' = 3x^2, v'' = 6x, v''' = 6$$

$$u = e^{4x}, u' = 4e^{4x}, u'' = 16e^{4x}, u''' = 64e^{4x}$$

$$u^n = 4^n e^{4x}$$

$$y^n = {}^n C_0 u^{n-0} v^0 + {}^n C_1 u^{n-1} v^1 + {}^n C_2 u^{n-2} v^2 + {}^n C_3 u^{n-3} v^3$$

$$= u^n v' + n u^{n-1} v' + \frac{n(n-1)}{2} u^{n-2} v'' + \frac{n(n-1)(n-2)}{3!} u^{n-3} v'''$$

$$= 4^n e^{4x} \cdot x^3 + n 4^{n-1} e^{4x} \cdot 3x^2 + \frac{n(n-1)}{2} 4^{n-2} e^{4x} \cdot 6x + n(n-1) 4^{n-3} e^{4x} \cdot 6$$

$$(n-2) \frac{4^{n-3} e^{4x} \cdot 6}{6x^2}$$

$$= 4^n e^{4x} x^3 + n 4^{n-1} e^{4x} \cdot 3x^2 + n(n-1) 4^{n-2} e^{4x} \cdot 3x + n(n-1)(n-2) 4^{n-3} e^{4x}$$

$$= 4^{n-3} e^{4x} [4^3 x^3 + n 4^2 \cdot 3x^2 + n(n-1) 4 \cdot 3x + n(n-1)(n-2)]$$

$$= 4^{n-3} e^{4x} [64x^3 + n 48x^2 + 12(n-1) + n(n-1)(n-2)]$$

$$y^3 = 4^{5-3} e^{4x} [64x^3 + (5x+8)x^2 + 12x \cdot 5(5-1)x + 5(5-1)(5-2)]$$

$$y^3 = 16 e^{4x} [64x^3 + 240x^2 + 240x + 60]$$

$$ii) \quad x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + y = 0$$

$$x^2 y'' + xy' + y = 0$$

$$w_1 = x^2 y''$$

$$u^0 = v = y''$$

$$v' = y''', \quad v'' = y^{(4)}$$

$$v^n = y^{n+2}$$

$$v = x^2, \quad v' = 2x, \quad v'' = 2, \quad v''' = 0$$

$$w_2 = xy'$$

$$v^0 = u = y'$$

$$v' = y'', \quad v'' = y'''$$

$$v^n = y^{n+1}$$

$$w_3 = y$$

$$v = y$$

$$v' = y', \quad v'' = y''$$

$$v^n = y^n$$

$$w_1^n = {}^n C_0 u^{n-0} v^0 + {}^n C_1 v^{n-1} v' + {}^n C_2 v^{n-2} v''$$

$$= v^n v + n v^{n-1} v' + \frac{n(n-1)}{2!} v^{n-2} v''$$

$$= y^{n+2} v + n y^{n+1} 2x + \frac{n(n-1)}{2} y^n$$

$$= y^{n+2} v + n y^{n+1} 2x + (n-1)n y^n$$

$$= y^n [y^2 x^2 + n y^2 x + n(n-1)]$$

$$w_2^n = {}^n C_0 v^{n-0} v^0 + {}^n C_1 v^{n-1} v' + {}^n C_2 v^{n-2} v'' + {}^n C_3 v^{n-3} v'''$$

$$= u^n v^0 + n u^{n-1} v' + \frac{(n-1)n}{2!} u^{n-1} \cdot 0$$

$$= y^{n+1} \cdot x + n y^n \cdot 1 + 0$$

$$= y^n (xy + n)$$

$$w_3^n = {}^n C_0 u^{n-0} v^0 + {}^n C_1 u^{n-1} v'$$

$$= u^n v^0 + 0$$

$$= y^n$$

$$x^2 y'' + xy' + y = 0$$

$$W_1 + W_2 + W_3 = 0$$

$$y^n [y^2 x^2 + n^2 xy + n(n-1)] + y^n (xy + n) + y^n = 0$$

$$\text{At } x = 0$$

$$y^n n(n-1) + y^n n + y^n = 0$$

$$n(n-1) + n + 1 = 0$$

$$y^n - y^n n(n-1) - ny^n$$

$$\text{at } n = 1$$

$$y = -0 - y'$$

$$y = -y'$$

$$\Rightarrow x^2 y^{n+2} + n^2 xy^{n+1} + n(n-1)y^n + xy^{n+1} + ny^n + y^n = 0$$

$$\Rightarrow x^2 y^{n+2} + xy^{n+1} (2n+1) + y^n (n^2 - n + n + 1) = 0$$

$$\Rightarrow x^2 y^{n+2} + (2n+1)xy^{n+1} + (n^2+1)y^n = 0$$