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16/ENG02/019.
Computer Engineering.
ENG 381 ENGINEERING MATHS III

i.) If $y = e^{x^2+x}$
show that
$y'' = y'(2x+1) + 2y$.
and hence prove that
$y^{(n+2)} = (2x+1)y^{(n+1)} + 2(n+1)y^n$.

ANSWER.

$y = e^{x^2+x}$.

$\ln y = x^2 + x$

Differentiating both sides.

$\frac{1}{y}\frac{dy}{dx} = 2x+1$

Multiply both sides by y,

$\frac{dy}{dx} = (2x+1)y$.

$\frac{d^2y}{dx^2} = U\frac{dv}{dx} + V\frac{du}{dx}$.

$V = 2x+1 \qquad dv/dx = 2$

$u = y \qquad du/dx = \frac{du}{dy}\cdot\frac{dy}{dx} = 1\cdot\frac{dy}{dx}$.

So that;

$\frac{d^2y}{dx^2} = (2x+1)\cdot 1\frac{dy}{dx} + 2y$.

$\frac{d^2y}{dx^2} = (2x+1)\frac{dy}{dx} + 2y$.

$y'' = y'(2x+1) + 2y$.

P 16/ENG02/019

$y'' = y'(2x+1) + 2y$

$y^{(2)} = y^{(1)}(2x+1) + 2y$

Differentiating $y^{(1)}(2x+1)$

let $v = 2x+1$ $\quad u = y'$

$v' = 2$ $\quad u^n = y^{(n+1)}$

$v^2 = 0$ $\quad u^{(n-1)} = y^n$

Recall from Leibnitz theorem

$U^n v + n u^{(n-1)} v' + \cancel{n(n-1) u^{(n-2)} v^2}$ but $v^2 = 0$

$= y^{(n+1)} \cdot (2x+1) + n(y^n) \cdot 2$

Differentiating $y^{(2)}$ we have; $y^{(n+2)}$

$(2y = 2y^n)$

$y^{(n+2)} = y^{(n+1)} \cdot (2x+1) + 2n y^n + 2y^n$

$y^{(n+2)} = (2x+1) y^{(n+1)} + 2y^n(n+1)$

Proved.

(2) Using Leibnitz theorem, given that

(i) $y = x^3 e^{4x}$, determine $y^{(5)}$

$u = e^{4x}$ $\quad v = x^3$

$u^n = 4^n e^{4x}$ $\quad v' = 3x^2$

$u^{(n-1)} = 4^{(n-1)} e^{4x}$ $\quad v^2 = 6x$

$u^{(n-2)} = 4^{(n-2)} e^{4x}$ $\quad v^3 = 6$

$u^{(n-3)} = 4^{(n-3)} e^{4x}$ $\quad v^4 = 0$

where $n = 5$

$4^5 e^{4x} = 1024 e^{4x}$

$u^{(n-1)} = 4^{(5-1)} e^{4x} = 256 e^{4x}$

$u^{(n-2)} = 4^{(5-2)} e^{4x} = 64 e^{4x}$

$u^{(n-3)} = 4^{(5-3)} e^{4x} = 16 e^{4x}$

16/ENG02/019

$$y^{(n)} = (1024e^{4x} \cdot x^3) + n(256e^{4x} \cdot 3x^2) + \frac{n(n-1)}{2!}(64e^{4x} \cdot 6x) + \frac{n(n-1)(n-2)}{3!}(16e^{4x}) \cdot 6$$

We have:

$$= (1024x^3 + 3840x^2 + 3840x + 960)e^{4x}.$$

[illegible]

$$y^{(n)} = e^{4x}(1024x^3 + 3840x^2 + 3840x + 960)$$

2(ii) $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

Show that $x^2y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2+1)y^n = 0$

for $x^2y'' + xy' + y = 0$

$u = y^{(2)}$ $\quad V = x^2$

$u^n = y^{(n+2)}$ $\quad V' = 2x$ $\Rightarrow y^{(n+2)}(x^2) + n(y^{n+1})2x + \frac{n(n-1)y^n(2)}{2!}$ $\quad$ Since $V^3 = 0$

$u^{(n-1)} = y^{(n+1)}$ $\quad V^2 = 2$

$u^{(n-2)} = y^n$ $\quad V^3 = 0$

for (xy')

$V = x$ $\quad u = y'$

$V' = 1$ $\quad u^n = y^{(n+1)}$

$V^2 = 0$ $\quad u^{(n-1)} = y^n$

Applying:

$$y^n = U^nV + nU^{(n-1)}V^{(1)} + \frac{n(n-1)}{2!}U^{(n-2)}V^2 + \cdots$$

V 0.

Since $V^2 = 0$

~~$y^n = y^{(n+2)}(x^2) + n(y^{n+1})(2x) + n(n-1)2y$~~

for (xy')

$$y^n = y^{(n+1)} \cdot x + ny^{(n)}$$

16/ENG02/019.

$y^n = x^2 y^{(n+2)} + 2xny^{(n+1)} + n(n-1)y^n + xy^{(n+1)} + ny^n - ey^n$

then

$y^n = x^2 y^{(n+2)} + xy^{(n+1)} \cdot (2n+1) + y^n(n^2 - n + n + 1) = 0$

$= x^2 y^{(n+2)} + xy^{(n+1)}(2n+1) + y^n(n^2+1) = 0$

$y^n = x^2 y^{(n+2)} + xy^{(n+1)}(2n+1) + y^n(n^2+1) = 0$