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161ENG04060
Electrical Engineering
ENG 381

Solution

1) $y = e^{x^2+x}$

$y' = (2x+1)e^{x^2+x}$

$u = 2x+1$

$\frac{du}{dx} = 2$

$v = e^{x^2+x}$

$\frac{dv}{dx} = (2x+1)e^{x^2+x}$

$y'' = u\frac{dv}{dx} + v\frac{du}{dx}$

$= (2x+1)(2x+1)e^{x^2+x} + e^{x^2+x}(2)$

$y'' = y'(2x+1) + 2y$

From Leibnitz theorem

$y'' - y'(2x+1) - 2y = 0$

$W_1 = y''$ $\qquad v = 1$

$u = y''$ $\qquad v' = 0$

$u^n = y^{n+2}$

$m_1^n = {}^nC_2 u^{n-0} v^2$

$= y^{n+2}$

$W_2 = y'(2x+1)$ $\qquad v' = 2$

$u = y'$ $\qquad v'' = 0$

$u^n = y^{n+1}$

$v_1^n = {}^nC_0 u^{n-0} v^0 + {}^nC_1 u^{n-1} v^1$

$= u^n v + n u^{n-1}$

$= y^{n+1}(2x+1) + n y^n \cdot 2$

$w_2 = 2y$

$u = y \ , \ v = 1 \ , \ v' = 0$

$u^n = y^n$

$w_2^n = {}^nC_0 u^{n-0} v$

$= 2y^n$

$y'' - y'(2x+1) - 2y = 0$

$y^{n+2} - y^{n+1}(2x+1) - 2ny^n - 2y^n = 0$

$y^{n+2} - y^{n+1}(2x+1) - 2y^n(n+1) = 0$

$y^{n+2} = y^{n+1}(2x+1) + 2y^n(n+1)$

2) i) $y = x^3 e^{4x}$

$v^0 = x^3 \ , \ v' = 3x^2 \ , \ v'' = 6x \ , \ v''' = 6$

$u = e^{4x} \ , \ u' = 4e^{4x} \ , \ u'' = 16e^{4x} \ , \ u''' = 64e^{4x}$

$u^n = 4^n e^{4x}$

$y^n = {}^nC_0 u^{n-0} v^0 + {}^nC_1 u^{n-1} v' + {}^nC_2 u^{n-2} v'' + {}^nC_3 u^{n-3} v'''$

$= u^n v' + n u^{n-1} v' + \frac{n(n-1)}{2} u^{n-2} v'' + \frac{n(n-1)(n-2)}{3!} v'''$

$= 4^n e^{4x} \cdot x^3 + n 4^{n-1} e^{4x} \cdot 3x^2 + \frac{n(n-1) 4^{n-2} e^{4x} \cdot \cancel{6}^{3} x}{\cancel{2}} + n(n-1)$

$(n-2) \frac{4^{n-3} e^{4x} \cdot \cancel{6}}{\cancel{6}}$

$= 4^n e^{4x} x^3 + n 4^{n-1} e^{4x} \cdot 3x^2 + n(n-1) 4^{n-2} e^{4x} \cdot 3x + n(n-1)(n-2) 4^{n-3} e^{4x}$

$= 4^{n-3} e^{4x} [4^3 x^3 + n 4^2 \cdot 3x^2 + n(n-1) 4 \cdot 3x + n(n-1)(n-2)]$

$= 4^{n-3} e^{4x} [64x^3 + n48x^2 + 12(n-1) + n(n-1)(n-2)]$

$y^3 = 4^{5-3} e^{4x} [64x^3 + (5 \times 48) x^2 + 12 \times 5(5-1)x + 5(5-1)(5-2)]$

$y^3 = 16 e^{4x} [64x^3 + 240x^2 + 240x + 60]$

ii) $x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + y = 0$

$x^2 y'' + xy' + y = 0$

$w_1 = x^2 y''$

$u^0 = v = y''$

$v' = y''', \ v'' = y^{iv}$

$v^n = y^{n+2}$

$v = x^2, \ v' = 2x, \ v'' = 2, \ v''' = 0$

$W_2 = xy'$

$v^0 = u = y'$

$v' = y'', \ v'' = y$

$v^n = y^{n+1}$

$W_3 = y$

$v = y$

$v' = y', \ v'' = y''$

$v^n = y^n$

$w_1^n = {}^nC_0 u^{n-0} v^0 + {}^nC_1 v^{n-1} v' + {}^nC_2 v^{n-2} v''$

$= v^n v + n v^{n-1} v' + \frac{n(n-1)}{2!} v^{n-2} v''$

$= y^{n+2} v + n y^{n+1} 2x + \frac{n(n-1)}{2} y^n$

$= y^{n+2} v + n y^{n+1} 2x + (n-1) n y^n$

$= y^n [y^2 x^2 + n y^2 x + n(n-1)]$

$W_2^n = {}^nC_0 v^{n-0} v^0 + {}^nC_1 v^{n-1} v' + {}^nC_2 v^{n-2} v'' + {}^nC_3 v^{n-3} v'''$

$= u^n v^0 + n u^{n-1} v' + \frac{(n-1)n}{2!} u^{n-1} \cdot 0$

$= y^{n+1} \cdot x + n y^n \cdot 1 + 0$

$= y^n (xy + n)$

$W_3^n = {}^nC_0 u^{n-0} v^0 + {}^nC_1 u^{n-1} v'$

$= u^n v^0 + 0$

$= y^n$

$$x^2 y'' + xy' + y = 0$$

$$W_1 + W_2 + W_3 = 0$$

$$y^n[y^2x^2 + n2xy + n(n-1)] + y^n(xy + n) + y^n = 0$$

At :- $x = 0$

$$y^n n(n-1) + y^n n + y^n = 0$$

$$n(n-1) + n + 1 = 0$$

$$y^n - - y^n n(n-1) - ny^n$$

at $n = 1$

$$y = -0 - y'$$

$$y = -y'$$

$$\Rightarrow x^2 y^{n+2} + n2xy^{n+1} + n(n-1)y^n + xy^{n+1} + ny^n + y^n = 0$$

$$\Rightarrow x'y^{n+2} + xy^{n+1}(2n+1) + y^n(n^2 - n + n + 1) = 0$$

$$\Rightarrow x'y^{n+2} + (2n+1)xy^{n+1} + (n^2+1)y^n = 0$$