VODINA EFEM
16/eng03/020
CIVIL ENGINEERING
ENG 381

1) If $y = e^{x^2+x}$
Show that $y'' = y'(2x+1)+2y$
and prove that $y^{(n+2)} = (2x+1)y^{(n-1)} + 2(n+1)y^{n}$

Solution
To find y'' we use product rule.

$u = 2x+1$

$v = e^{x^2+x}$

$$y'' = u\frac{dv}{dx} + v\frac{du}{dx}.$$

$$dy/dx = (2x+1)e^{x^2+x}$$

$$du/dx = 2$$

$$y'' = (2x+1)(2x+1)e^{x^2+x} + 2(e^{x^2+x})$$

Since $y' = (2x+1)e^{x^2+x}$ and $y = e^{x^2+x}$

$$y'' = y'(2x+1)+2y$$

$$y'' - y'(2x+1) - 2y = 0.$$

Let $w_1 = y''$

$u^{(0)} = y''$, $u^{(1)} = y'''$, $u^{(11)} = y^{1V}$.

$u^{(n)} = y^{(n+2)}$, $w_1^{(n)} = y^{(n+2)}$

Let $w_2 = y'(2x+1)$

$u^{(0)} = y'$, $u^{(1)} = y''$, $u^{(11)} = y'''$, $u^{(n)} = y^{(n+1)}$

$v^{(0)} = 2x+1$, $v^{(1)} = 2$, $v^{(11)} = 0$.

$w_2 = y^{(n+1)} . 2x+1 + ny^{n} . 2$

$= y^{n+1}(2x+1) + 2ny^{n}$

$w_3 = 2y$

$u = y$, $u^{n} = y^{n}$

$u^{(n)} = y^{(n)}2$

$w_3 = 2y^n$

$y^{(n+2)} - (y^{(n+1)}(2x+1) + 2ny^n) - 2y^n = 0$

$y^{(n+2)} = y^{(n+1)}(2x+1) + 2ny^n + 2y^n$

2) $y = x^3 e^{4x}$, determine y^5.

$v = x^3$ $\quad u = e^{4x}$

$v^{(0)} = x^3$ $\quad v^{(1)} = 3x^2$, $\quad v^{(11)} = 6x$.

$u^{(0)} = e^{4x}$ $\quad u^{(1)} = 4e^{4x}$, $\quad u^{(11)} = 16e^{4x}$

$u^{111} = 64e^{4x}$

$u^{(n)} = 4^n \cdot e^{4x}$

$$y^{(n)} = u^{(n)} v^{(0)} + n u^{(n-1)} v^{(1)} + \frac{n(n-1)}{2} u^{n-2} v^{(2)}$$

$$y^{(n)} = 4^n e^{4x} \cdot x^3 + n 4^{n-1} e^{4x} \cdot 3x + \frac{n(n-1)}{2} 4^{n-2} e^{4x} \cdot 6x$$

$$+ \frac{n(n-1)(n-2)6}{6} \cdot 4^{n-3} \cdot e^{4x}$$

$y^{(n)} = 4^{n-3} \cdot e^{4x}(64x^3 + 48nx^2 + 12n(n-1)x + n(n-1)(n-2))$

$y^{(5)} = 4^{5-3} \cdot e^{4x}(64x^3 + 48(5)x^2 + 12 \cdot 5(5-1)x + 5(5+1)(5-2))$.

$y^{(5)} = 4^2 \cdot e^{4x}(64x^3 + 240x^2 + 240x + 60)$

$y^{(5)} = 16e^{4x}(64x^3 + 240x^2 + 240x + 60)$.

3) $\frac{x^2 d^2y}{dx^2} + \frac{x dy}{dx} + y = 0$.

Show that $x^2 y^{(n-2)} + (2n+1)xy^{(n+1)} + (n^2+1)y^n = 0$.

$x^2 y'' + xy' + y = 0$.

Let $x^2 y'' = w_1$

$xy' = w_2$

$y = w_3$

[redacted]

$w_1^{(n)} = x^2 y''$

$v^{(0)} = x^2, \quad v^{(1)} = 2x, \quad v'' = 2.$

$u^{(0)} = y'', \quad u^{(1)} = y''', \quad u^{(11)} = y^{IV}, \quad u^{(n)} = y^{(n+2)}$

$w_1^{(n)} = y^{(n+2)} x^2 + n y^{n+1} \cdot 2x + \frac{n(n-1) y^{(n)}}{2} \cdot 2.$

$w_1^{(n)} = x^2 y^{(n+2)} + 2xny^{(n+1)} + n(n-1)y^n.$

$w_2^{(n)} = xy'$

$u^{(0)} = y', \quad u^{(1)} = y'', \quad u''' = y''', \quad u^n = y^{(n+1)}$

$v^{(0)} = x, \quad v^{(1)} = 1$

$w_2^{(n)} = y^{(n+1)} \cdot x + n y^n \cdot 1$

$w_2^{(n)} = xy^{(n+1)} + ny^n$

$w_3 = y.$

$u^{(0)} = y, \quad u^{(1)} = y', \quad u^{(n)} = y^{(n)}$

$v^{(0)} = 1 \quad v'' = 1.$

$w_3^{(1)} = y^{(n)} \cdot 1 + u^{(n-1)}(0)$

$w_3^{(n)} = y^n.$

$x^2 y^{(n+2)} + 2xny^{(n+1)} + n(n-1)y^n + xy^{(n+1)} + ny^n + y^n = 0$

$x^2 y^{(n+2)} + 2xny^{(n+1)} + xy^{(n+1)} + n(n-1)y^n + ny^n + y^n = 0$

$x^2 y^{(n+2)} + xy^{(n+1)}(2n+1) + y^n(n(n-1)+n+1) = 0.$

$x^2 y^{(n+2)} + xy^{(n+1)}(2n+1) + y^n(n^2-n+n+1) = 0$

$x^2 y^{(n+2)} + xy^{(n+1)}(2n+1) + y^n(n^2+1) = 0.$