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MECHATRONICS

16/ENG05/006

(i) $y = e^{x^2+x}$

$y' = (2x+1)e^{x^2+x}$

$u = 2x+1$, $v = e^{x^2+x}$

$\frac{du}{dx} = 2$, $\frac{dv}{dx} = (2x+1)e^{x^2+x}$

$y'' = u\frac{dv}{dx} + v\frac{du}{dx}$

$= (2x+1)(2x+1)e^{x^2+x} + e^{x^2+x}(2)$

$y'' = y'(2x+1) + 2y$

$y'' = y'(2x+1) + 2y$

$y'' - y'(2x+1) - 2y = 0$

$W_1 = y''$

$u = y''$ $\quad v = 1$

$u^n = y^{n+2}$ $\quad v' = 0$

$W_1^n = {}^nC_0 u^{n-0} v^0 + {}^nC_1 u^{n-1} v'$

$= y^{n+2} + 0$

$= y^{n+2}$

$W = y'(2x+1)$

$u = y'$ $\quad v = 2x+1$

$u^n = y^{n+1}$ $\quad v' = 2$ $\quad v'' = 0$

$W_2^n = {}^nC_0 u^{n-0} v^0 + {}^nC_1 u^{n-1} v'$

$= u^n v + n u^{n-1} v'$

$= y^{n+1}(2x+1) + 2ny^n$

$W_3 = 2y$

$u = y$ $\quad v = 2$ $\quad v' = 0$

$u' = y$

$u^n = y^n$

$W_3^n = {}^nC_0\, u^{n-0} v^0$

$= u^n v$

$W_3^n = 2y^n$

$y'' - y'(2x+1) - 2y = 0$

$y^{n+2} - (y^{n+1}(2x+1) + 2ny^n) - 2y = 0$

$y^{n+2} - y^{n+1}(2x+1) + 2ny^n) - 2y = 0$

$y^{n+2} - y^{n+1}(2x+1) - 2y^n(n+1) = 0$

$y^{n+2} = y^{n+1}(2x+1) + 2y^n(n+1)$. [PROVEN].

2;) $y = x^3 e^{4x}$:

$$y^5 = u^5 v + 5u^{(5-1)} v' + \frac{5(5-1)u^{(5-2)}}{2!} v^{(2)} + \frac{5(5-1)(5-2)}{3!} u^3 v^3$$

$$+ \frac{5(5-1)(5-2)(5-3)u' v^4}{4!} + uv^{(5)}$$

$$y^5 = u^5 v + 5u^4 v' + \frac{20 \cdot u^3 v^2}{2!} + 60u^2 v^3 + \frac{120u' v^4}{4!} + uv^5$$

$$y^5 = u^5 v + 5u^4 v' + 10u^3 v^2 + 10u^2 v^3 + 5u' v^4 + uv^5$$

When $u = e^{4x}$ | $v = x^3$

$u^5 = 4^5 e^{4x}$ | $v' = 3x^2$

$u^5 = 1024\, e^{4x}$ | $v^2 = 6x$

$u^4 = 4^4 e^{4x} = 256 e^{4x}$ | $v^3 = 6$

$u^3 = 4^3 e^{4x} = 64 e^{4x}$ | $v^4 = 0$

$u^2 = 4^2 e^{4x} = 16 e^{4x}$ | $v^5 = 0$

$u' = 4' e^{4x} = 4e^{4x}$

$$y^{(5)} = 1024\, e^{4x} \cdot x^3 + 5 \cdot 256 \cdot e^{4x} \cdot 3x^2 + 10 \cdot 64\, e^{4x} \cdot 6x +$$
$$10 \cdot 16\, e^{4x} \cdot 6 + 5 \cdot 16\, e^{4x} \cdot 0 + e^{4x} \cdot 0$$

$$y^5 = 64e^{4x}\{16x^3 + 60x^2 + 6x + 15\}$$

9(i) $x^2\dfrac{d^2y}{dx^2} + x\dfrac{dy}{dx} + y = 0$

This equation can be rewritten as

$x^2y'' + xy' + y = 0$

Taking each term and differentiating n times

Using Leibnitz theorem

If $W_1 = x^2y''$

Let $u = y^{(2)}$ $\qquad V = x^2$

$u^{(n)} = y^{(n+2)}$ $\qquad V^{(1)} = 2x$

$u^{(n-1)} = y^{(n+1)}$ $\qquad V^{(2)} = 2$

$u^{(n-2)} = y^n$ $\qquad V^3 = 0$

$\therefore W_1^{(n)}$

From $y^{(n)} = u^nV + nV^{(n-1)}V^{(1)} + \dfrac{n(n-1)u^{(n-2)}V^2}{2!} + \dots$

$W_1^{(n)} = y^{(n+2)}x^2 + n\cdot y^{(n+1)}\dfrac{2x + n(n-1)y^n 2}{2} + 0$

$w_1^n = x^2y^{(n+2)} + 2xny^{(n+1)} + n(n-1)y^{(n)}$

If $W_2 = xy'$

Let $u = y^{(1)}$ $\qquad V = x$

$u^{(n)} = y^{(n+1)}$ $\qquad V^{(1)} = 1$

$u^{(n-1)} = y^{(n)}$ $\qquad V^{(2)} = 0$

$W_2^n = y^{(n+1)}\cdot x + ny^{(n)}\cdot 1 + 0$

If $W_3 = y$

$u = y$

$u^{(n)} = y^{(n)}$

$\therefore W_3^{(n)} = y^n$

Therefore $W_1^{(n)} + W_2^n + W_3^n = 0$

That is

$$x^2 y^{n+2} + 2xny^{(n+1)} + n(n-1)y^n + y^{(n+1)}x + ny^n + y^n = 0$$

$$x^2 y^{(n+2)} + xy^{(n+1)}(2n+1) + y^n(n^2 - n + n + 1) = 0$$

$$x^2 y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2+1)y^n = 0$$