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1) If $y = e^{x^2+x}$
show that

$$y'' = y'(2x+1) + 2y$$

and hence prove that; $y^{(n+2)} = (2x+1)y^{(n+1)} + 2(n+1)y^n$

Answer

$$y = e^{x^2+x} \quad \text{———} \quad (1)$$

using chain rule — $\frac{\delta y}{\delta x} = \frac{\delta y}{\delta u} \times \frac{\delta u}{\delta x}$

$u = x^2 + x$

$y = e^u$

$\frac{\delta y}{\delta u} = e^u \qquad \frac{\delta u}{\delta x} = 2x+1$

$$\frac{\delta y}{\delta x} = e^u(2x+1) = e^{x^2+x}(2x+1) \quad \text{———} \quad (2)$$

$\cancel{\frac{\delta^2 y}{\delta x^2}}$ $\frac{\delta y}{\delta x} = U \frac{\delta v}{\delta x} + V \frac{\delta u}{\delta x}$ (from product rule)

let $u = e^{x^2+x} \qquad u = e^y$

and $y = x^2 + x$

$\frac{\delta u}{\delta y} = e^y \qquad \frac{\delta y}{\delta x} = 2x+1 \qquad \frac{\delta u}{\delta x} = e^y(2x+1)$

$$\frac{\delta y}{\delta x} = (2x+1)e^{x^2+x}$$

let $v = 2x+1$

$$\frac{\delta v}{\delta x} = 2$$

$$\frac{\delta^2 y}{\delta x^2} = \left(e^{x^2+x}\right) 2 + (2x+1)(2x+1)e^{x^2+x}$$

$$= 2\left(e^{x^2+x}\right) + (2x+1)(2x+1)e^{x^2+x}$$

but recall that

$y = e^{x^2+x}$ — from equation 1

and $y' = (2x+1)e^{x^2+x}$

$y'' = 2y + (2x+1)y'$

$y'' = y'(2x+1) + 2y$ ——— (3)

shown

from ——— (3) $y'' = y'(2x+1) + 2y$

$y^2 = y^{(1)}(2x+1) + 2y$

let $w_1 = y^{(2)}$

$w_2 = y^{(1)}(2x+1)$

$w_3 = 2y$

a) $w_1 = y^{(2)}$

$u = y^{(2)}$, $v = 1$

$u^n = y^{n+2}$ $v' = 0$

Hence ; $w_1^n = v^n v^0 + {}^nC_1 u^{n-1} v' + {}^nC_2 u^{n-2} v^2 + \ldots$

$w_1^n = u^n v^0 + n u^{n-1} v' + \dfrac{n(n-1)}{2} u^{n-2} v^2$

$w_1^n = y^{n+2} \cdot 1 + n y^{n-1+2} \cdot 0$

$w_1^n = y^{n+2}$

b) $w_2 = y^{(1)}(2x+1)$

$u = y'$ $v = 2x+1$

$u' = y''$ $v' = 2$

$v'' = 0$

$u^n = y^{n+1}$

$w_2^n = u^n v^n + n u^{n-1} v' + \dfrac{n(n-1)}{2} u^{n-2} v^2$

$w_2^n = y^{n+1}(2x+1) + n(y^{n+1-1})2 + \dfrac{n(n-1)}{2} y^{n-2+1} \cdot 0$

$= (2x+1)y^{n+1} + 2ny^n$

c) $w_3 = 2y$

$u = y$, $v = 2$

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$u' = y', \quad v' = 0$

$u^n = y^n$

$w_3^n = u^n v^3 + n u^{n-1} v' + \frac{n(n-1)}{2} u^{n-2} v^2$

$w_3^n = y^n . 2 + n y^{n-1} . 0$

$w_3^n = 2y^n$

$w_1^n = w_2^n + w_3^n$

$\underline{y^{(n+2)} = (2x+1) y^{n+1} + 2(n+1) y^n}$ proven

Question 2

$y = x^3 e^{4x}$

$v = x^3, \quad u = e^{4x}$

$v' = 3x^2 \quad u' = 4e^{4x}$

$v'' = 6x \quad u'' = 16e^{4x} \quad \therefore u^n = 4^n e^{4x}$

$v''' = 6 \quad u''' = 64e^{4x}$

$y^n = {}^nC_0 u^n v^0 + {}^nC_1 u^{n-1} v' + {}^nC_2 u^{n-2} v^2 + {}^nC_3 u^{n-3} v^3$

$y^n = u^n v^0 + n u^{n-1} v'$

$y^n = u^n v^0 + n u^{n-1} v' + \frac{n(n-1)}{2!} v^2 u^{n-2} + \frac{n(n-1)(n-2) u^{n-3}}{3!} v^3$

$\cancel{y^n = u^n v^0 + n u^{n-1} v' +}$

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where $n=5$

$$y^{(5)} = (4^5 e^{4x} \cdot x^3) \cdot (5 \cdot 4^{5-1} e^{4x} \cdot 3x^2) + \frac{5(5-1)4^{5-2}e^{4x} \cdot 6x}{2\times 1}$$

$$+ \frac{5(5-1)(5-2)4^{5-3}e^{4x} \cdot 6}{3\times 2\times 1}$$

$$y^{(5)} = 1024e^{4x}x^3 + 3840e^{4x}x^2 + 3840e^{4x}x + 960e^{4x}$$

$$y^{(5)} = 64e^{4x}(16x^3 + 60x^2 + 15)$$

ii) $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

show that $x^2 y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2+1)y^{(n)} = 0$

$$x^2y'' + xy' + y = 0$$

a) let $w_1 = x^2y''$ so that $w_1{}^n + w_2{}^n + w_3{}^n = 0$

$v = x^2 \qquad u = y''$

$v' = 2x \qquad u' = y'''$

$v'' = 2 \qquad u'' = y^{iv}$

$u^n = y^{n+2}$

$$w_1{}^n = u^n v^0 + nu^{n-1}v' + \frac{n(n-1)}{2}u^{n-2}v^2$$

$$w_1{}^n = y^{n+2}x^2 + n(y^{n+2-1}) \cdot 2x + \frac{n(n-1)}{2}y^{(n+2-2)} \cdot 2 + 0$$

$$+ [y^{n+1} \cdot 2x + ny^{(n)} \cdot 2 + 0] - y^n = 0$$

$$\cancel{w_1^n = x^2 (y^{n+2}) + n(y^{n+2-1})\, 2x +} \; n(n-1)\, y^n$$

$$\cancel{= x^2 y\,\epsilon}$$

$$= x^2 y^{(n+2)} + 2xny^{(n+1)} + n(n-1)y^n + 2xy^{(n+1)} + 2ny^{(n)} + y^{(n)}$$

$$= x^2 y^{(n+2)} + 2(n+1)xy^{(n+1)} + (n^2 - n + 2n + 1)y^{(n)} = 0$$

$$= x^2 y^{(n+2)} + 2(n+1)xy^{(n+1)} + (n^2+1)y^{(n)} = 0$$