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ENG 381 - Assigment 2.

1) If $y = e^{x^2+x}$
show that
$y'' = y'(2x+1) + 2y$
and hence prove that
$y^{(x+2)} = (2x+1)y^{(n+1)} + 2(n+1)y^{n}$

Solution.

$y = e^{x^2+x}$

$\ln y = x^2 + x$

Differentiating Both sides

$\frac{1}{y}\frac{dy}{dx} = 2x + 1$

Multiply both sides by y

$\frac{dy}{dx} = (2x+1)y$

$\frac{d^2y}{dx^2} = u\frac{dv}{dx} + v\frac{du}{dx}$

$v = 2x+1$ $\quad$ $dv/dx = 2$

$u = y$ $\quad$ $du/dx = \frac{du}{dy}\cdot\frac{dy}{dx} = 1.\frac{dy}{dx}$

So that,

$\frac{d^2y}{dx^2} = (2x+1).1\frac{dy}{dx} + 2y$

$\frac{d^2y}{dx^2} = (2x+1)\frac{dy}{dx} + 2y$

$y'' = y'(2x+1) + 2y$

$y'' = y'(2x+1) + 2y$

$y^{(2)} = y^{(1)}(2x+1) + 2y$

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Differentiating $y^{(1)}(2x+1)$

Let $V = 2x+1$ $\quad u = y'$

$V' = 2$ $\quad u^n = y^{(n+1)}$

$V'' = 0$ $\quad u^{(n-1)} = y^n.$

Recall from Leibnitz theorem.

$u^n v + n u^{(n-1)} v' + \cancel{\frac{n(n-1) u^{(n-2)} v^2}{2!}} \to 0$ But $v^2 = 0$.

$= y^{(n+1)}.(2x+1) + n(y^n).2.$

Differentiating $y^{(n)}$ we have, $y^{(n+2)}$

$y^{(n+2)} = y^{(n+1)}.(2x+1) + 2ny^n + 2y^n$

$y^{(n+2)} = (2x+1)y^{(n+1)} + 2y^n(n+1) \Rightarrow$ Proved.

2) Using Leibnitz theorem, given that

(c) $y = x^3 e^{4x}$, determine $y^{(5)}$

$u = e^{4x}$ $\quad V = x^3$

$u^n = 4^n e^{4x}$ $\quad v' = 3x^2$

$u^{(n-1)} = 4^{(n-1)} e^{4x}$ $\quad v'' = 6x$

$u^{(n-2)} = 4^{(n-2)} e^{4x}$ $\quad v''' = 6$

$u^{(n-3)} = 4^{(n-3)} e^{4x}$ $\quad v^{iv} = 0.$

Where $n = 5$

$4^5 e^{4x} = 1024 e^{4x}$

$u^{(n-1)} = 4^{(5-1)} e^{4x} = 256 e^{4x}$

$u^{(n-2)} = 4^{(5-2)} e^{4x} = 64 e^{4x}$

$u^{(n-3)} = 4^{(5-3)} e^{4x} = 16 e^{4x}$

$y^{(5)} = (1024 e^{4x}.x^3) + n(256 e^{4x}.3x^2) + \frac{n(n-1)\,64 e^{4x}.6x}{2!}$

$+ \frac{n(n-1)(n-2)(16e^{4x}).6}{3!}$

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We have;

$= (1024x^3 + 13840x^2 + 3840x + 960)e^{4x}$.

$y^{(5)} = e^{4x}(1024x^3 + 3840x^2 + 3840x + 960)$

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$x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

Show that $x^2y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2+1)y^{n} = 0$

$x^2y'' + xy' + y = 0$

$u = y^{(2)}$ $\quad v = x^2$

$u^n = y^{(n+2)}$ $\quad v' = 2x$

$u^{(n-1)} = y^{(n+1)}$ $\quad v'' = 2$

$u^{(n-2)} = y^n$ $\quad v''' = 0$

$y^{(n+2)}(x^2) + n(y^{n+1})2x + \frac{n(n-1)y^n(2)}{2!}$

for (xy')

$v = x$ $\quad u = y'$

$v' = 1$ $\quad u^n = y^{(n+1)}$

$v'' = 0$ $\quad u^{(n-1)} = y^n$.

Applying

$y^n = u^n v + nu^{(n-1)}v^{(1)} + \frac{n(n-1)u^{(n-2)}v^2}{2!} + \dots$

Since $v^2 = 0$

$y^n = y^{(n+1)} \cdot x + n^{(n)}$

$y^n = x^2 y^{(n+2)} + 2xny^{(n+1)} + n(n-1)y^n + ny^{(n+1)} + ny^n + y^n$.

~~then~~

$y^n = x^2y^{(n+2)} + xy^{(n+1)}(2n+1) + y^n(n^2 - n + n + 1) = 0$

$= x^2y^{(n+2)} + xy^{(n+1)}(2n+1) + y^n(n^2+1) = 0$

$y^n = x^2y^{(n+2)} + xy^{(n+1)}(2n+1) + y^n(n^2+1) = 0$.