Nduka Nnaemeka
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COMPUTER ENGR
ENG381

1) $y = e^{x^2+x}$

$y' = (2x+1)e^{x^2+x}$

$u = 2x+1$ $\qquad$ $v = e^{x^2+x}$

$\frac{du}{dx} = 2$ $\qquad$ $\frac{dv}{dx} = (2x+1)e^{x^2+x}$

$y'' = u\frac{dv}{dx} + v\frac{du}{dx}$

$= (2x+1)(2x+1)e^{x^2+x} + e^{x^2+x}(2)$

$y'' = y'(2x+1) + 2y$

From leibnitz theorem

$y'' - y'(2x+1) - 2y = 0$

$w_1 = y''$ $\qquad$ $v = 1$ $\qquad$ $u^n = y^{n+2}$

$u = y''$ $\qquad$ $v' = 0$ $\qquad$ $w_1^n = {}^nC_0 u^{n-0}v^0$

$= y^{n+2}$

$w_2 = y'(2x+1)$ $\qquad$ $v = 2x+1$

$u = y'$ $\qquad$ $v' = 2$

$u^n = y^{n+1}$ $\qquad$ $v_1^n = {}^nC_0 u^{n-0}v^0 + {}^nC_1 u^{n-1}v^1$

$v'' = 0$

$= u^n v + nu^{n-1}$

$= y^{n+1}(2x+1) + ny^n \cdot 2$

$w_3 = 2y$

$u = y$, $\quad v = 2$, $\quad v' = 0$

$$y'' - y'(2x+1) - 2y = 0$$
$$y^{n+2} - y^{n+1}(2x+1) - 2ny^n - 2y^n = 0$$
$$y^{n+2} - y^{n+1}(2x+1) - 2y^n(n+1) = 0$$

$$y^{n+2} = y^{n+1}(2x+1) + 2y^n(n+1)$$

2. $y = x^3 e^{4x}$

$v^0 = x^3$, $v' = 3x^2$, $v'' = 6x$, $v''' = 6$

$u = e^{4x}$, $u' = 4e^{4x}$, $u'' = 16e^{4x}$, $u''' = 64e^{4x}$

$u^n = 4^n e^{4x}$

$$y^n = {}^nC_0 u^{n-0} v^0 + {}^nC_1 u^{n-1} v' + {}^nC_2 u^{n-2} v'' + {}^nC_3 u^{n-3} v'''$$

$$= u^n v' + n u^{n-1} v' + \frac{n(n-1)}{2!} u^{n-2} v'' + \frac{n(n-1)(n-2)}{3!} v'''$$

$$= 4^n e^{4x} \cdot x^3 + n 4^{n-1} e^{4x} \cdot 3x^2 + \frac{n(n-1)\, 4^{n-2} e^{4x} \cdot \cancel{6}^{3} x}{\cancel{2}} + \frac{\cancel{6} n(n-1)(n-2) 4^{n-3} e^{4x} \cdot \cancel{6}}{\cancel{3} \times 2}$$

$$= 4^n e^{4x} x^3 + n 4^{n-1} e^{4x} \cdot 3x^2 + n(n-1) 4^{n-2} e^{4x} \cdot 3x + n(n-1)(n-2) 4^{n-3} e^{4x}$$

$$= 4^{n-3} e^{4x} \left[4^3 x^3 + n 4^2 \cdot 3x^2 + n(n-1) 4 \cdot 3x + n(n-1)(n-2) \right]$$
$$= 4^{n-3} e^{4x} \left[64x^3 + n 48x^2 + 12(n-1)x + n(n-1)(n-2) \right]$$

$$y^5 = 4^{5-3} e^{4x} \left[64x^3 + (5 \times 48) x^2 + 12 \times 5(5-1)x + 5(5-1)(5-2) \right]$$

$$y^5 = 16 e^{4x} \left[64x^3 + 240x^2 + 240x + 60 \right]$$

ii. $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

$x^2y'' + xy' + y = 0$

$W_1 = x^2y''$ $\qquad$ $V = x^2$, $V' = 2x$, $V'' = 2$

$U^0 = U = y''$ $\qquad$ $V''' = 0$

$U' = y'''$, $U'' = y^{iv}$

$U^n = y^{n+2}$

$W_2 = xy'$

$U^0 = U = y'$ $\qquad$ $V = x$, $V' = 1$, $V'' = 0$

$U' = y''$, $U'' = y'''$

$U^n = y^{n+1}$

$W_3 = y$

$U = y$ $\qquad$ $V = 1$, $V' = 0$

$U' = y'$, $U'' = y''$

$U^n = y^n$

$W_1^n = {}^nC_0U^{n-0}V^0 + {}^nC_1U^{n-1}V' + {}^nC_2U^{n-2}V''$

$= U^nV + nU^{n-1}V' + \frac{n(n-1)}{2!}U^{n-2}V''$

$= y^{n+2}V + ny^{n+1}2x + \frac{n(n-1)}{\cancel{2}}y^n\cancel{2}$

$= y^{n+2}V + ny^{n+1}2x + (n-1)n\ y^n$

$= y^n[y^2x^2 + ny2x + n(n-1)]$

$W_2^n = {}^nC_0U^{n-0}V^0 + {}^nC_1U^{n-1}V' + {}^nC_2U^{n-2}V'' + {}^nC_3U^{n-3}V'''$

$= U^nV^0 + nU^{n-1}V' + \frac{(n-1)n}{2!}U^{n-2}\cdot 0$

$= y^{n+1}\cdot x + \cancel{nxy}\ ny^n\cdot 1 + 0$

$= y^n(xy + n)$

$W_3^n = {}^nC_0U^{n-0}V^0 + {}^nC_1U^{n-1}V'$

$= U^nV^0 + 0$

$= y^n$

$$x^2 y'' + xy' + y = 0$$

$$w_1 + w_2 + w_3 = 0$$

$$y^n [y^2 x^2 + n\, 2xy + n(n-1)] + y^n (xy + n) + y^n = 0$$

At : $x = 0$

$$\cancel{y^n(x)} \quad y^n n(n-1) + y^n n + y^n = 0$$

$$n(n-1) + n + 1 = 0$$

$$y^n = -y^n (n-1) - ny^n$$

at $n = 1$

$$y = -0 - y'$$

$$y = -y'$$

$$\Rightarrow x^2 y^{n+2} + n\,2x y^{n+1} + n(n-1) y^n + x y^{n+1} + n y^n + y^n = 0$$

$$\Rightarrow x' y^{n+2} + x y^{n+1} (2n+1) + y^n (n^2 - n + n + 1) = 0$$

$$\Rightarrow x' y^{n+2} + (2n+1) x y^{n+1} + (n^2 + 1) y^n = 0$$