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1) If $y = e^{x^2+x}$

$\frac{dy}{dx} = y' \quad 2x+1\,(e^{x^2+x})$

$y'' = \frac{d^2y}{dx^2} = (2x+1)(e^{x^2+x})(2x+1) + (e^{x^2+x})(2)$

Putting (1) and (2) into (3)

$y'' = y'(2x+1) + y(2)$

$y'' = y'(2x+1) + 2y$

$y'' - y'(2x+1) - 2y = 0$

$\omega_1 = y''$

$U^0 = y''$, $U' = y''$, $U^n = y^{n+2}$

$V^0 = 1$, $V' = 0$

$\omega_1^{(n)} = U^n V^0 + n U^{(n-1)} V$

$\omega_1^n = y^{n+2}(1) + n y^{(n+1)}(0) = y^{n+2}$

$\omega_{11} = y'(2x+1)$

$V^0 = 2x+1$, $V' = 2$

$U^0 = y'$, $U' = y''$, $U^n = y^{n+1}$

$\omega_{11}^{(n)} = U^n V^0 + n U^{n-1} V'$

$\omega_{11}^{(n)} = y^{n+1}(2x+1) + n y^n (2)$

$\omega_{11}^{(n)} = (2x+1) y^{n+1} + 2ny^n$

$\omega_{111} = y$

$U^0 = y^0 \qquad U^n = y^n$

$V^0 = 1$

$\omega_{111}^{(n)} = y^n V^0 = y^n(1) = y^n$

$\omega_{i}^{(n)} + \omega_{ii}^{(n)} + \omega_{iii}^{(n)} = 0$ (ii)

$y^{n+2} - (2x+1)y^{n+1} - 2ny^{n} + 2y^{n} = 0$

$y^{n+2} - (2x+1)y^{n+1} + y^{n}(2n-2) = 0$

$y^{n+2} - (2x+1)y^{n+1} - 2(n+1)y^{n} = 0$

$y^{n+2} = (2x+1)y^{n+1} + 2(n+1)y^{n}$

2) Using the leibnitz theorem, given that

1) $y = x^3 e^{4x}$, determine $y^{(5)}$

Solution

$y = x^3 e^{4x}$, $y^{(5)} \to$

$U^0 = e^{4x}$, $U' = 4e^{4x}$, $U'' = 16e^{4x}$, $U''' = 64e^{4x}$

$U^{iv} = 4^4 e^{4x}$, $U^n = 4^n e^{4x}$

$V^0 = x^3$, $V' = 3x^2$, $V'' = 6x$, $V''' = 6$

$y^n = {}^nC_0 U^5 V^0 + {}^nC_1 U^{n-1} V' + {}^nC_2 U^{n-2} V^2 + \dots + {}^nC_n U^0 V^n$

$y^5 = {}^5C_0 U^5 V^0 + {}^5C_1 U^4 V' + {}^5C_2 U^3 V^2 + {}^5C_3 U^2 V^3 + {}^5C_4 U' V^4 + {}^5C_5 U^0 V^5$

$y^5 = U^5 V^0 + 5U^4 V' + 10U^3 V^2 + 10U^2 V^3 + 5U' V^4 + V^5$

$y^5 = 4^5 e^{4x} x^3 + 5 \cdot 4^4 e^{4x} \cdot 3x^2 + 10 \cdot 4^3 e^{4x}\, 6x + 10\, 4^2 e^{4x} \cdot 6 + 5 \cdot 4' e^{4x}(0)$

$y^5 = 4^5 x^3 e^{4x} + 15 \cdot 4^4 x^2 e^{4x} + 60 \cdot 4^3 x e^{4x} + 60 \cdot 4^2 e^{4x}$

$y^5 = 1024x^3 e^{4x} + 3840x^2 e^{4x} + 3840x e^{4x} + 960e^{4x}$

$y^5 = e^{4x}[1024x^3 + 3840x^2 + 3840x + 960]$

$x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$ Show that

$x^2 y^{(n+2)} + (2n+1)xy^{(n+1)} + (n^2+1)y^n = 0$

Solution

$x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

$x^2 y'' + xy' + y^0 = 0$

$w_1 = x^2 y''$

$v^0 = x^2, \quad v' = 2x, \quad v'' = 2$

$u^0 = y^2, \quad u' = y^3, \quad u'' = y^4, \quad u^n = y^{n+2}$

$w^{(n)} = u^n v^0 + n u^{n-1} v' + \frac{n(n-1)}{2!} u^{n-2} v''$

$w_1^{(n)} = y^{(n+2)}(x^2) + ny^{n+1}(2x) + \frac{n(n-1)}{\cancel{2}} y^n (2)$

$w_1^{(n)} = x^2 y^{(n+2)} + 2xny^{(n+1)} + n(n-1)y^n$

$w_{11} = xy'$

$v^0 = x, \quad v' = 1$

$u^0 = y', \quad u' = y^2, \quad u^n = y^{n+1}$

$w_{11}^{(n)} = u^n v^0 + n u^{n-1} v'$

$w_{11}^{(n)} = y^{n+1}(x) + n(y^n)(1) = xy^{n+1} + ny^n$

$w_{111} = y^0$

$u^0 = y^0, \quad u^n = y^n$

$w_{111}^{(n)} = y^n$

$w_1^{(n)} + w_{11}^{(n)} + w_{111}^{(n)} = 0$

$x^2 y^{(n+2)} + 2xny^{(n+1)} + n(n-1)y^n + xy^{n+1} + ny^n + y^n = 0$

$x^2 y^{(n+2)} + y^{(n+1)}[2n+1)x + y^n(n(n-1) + n + 1) = 0$

$x^2 y^{(n+2)} + y^{(n+1)}[2n+1)x + y^n(n^2 - n + n + 1) = 0$

$x^2 y^{(n+2)} + y^{n+1}[2n+1]x + y^n[n^2+1] = 0$

$$x^2 y^{n+2} = -y^{(n+1)} [2n+1] x - y^n (n^2+1)$$

$$x^2 y^{(n+2)} + x y^{(n+1)} (2n+1) + (n^2+1) y^n = 0$$

$$x^2 y^{(n+2)} + (2n+1) x y^{(n+1)} + (n^2+1) y^n = 0 .$$